A Letters Journal Exploring
the Frontiers of Physics

OFFPRINT

Preferential detachment in broadcast signaling networks: Connectivity and cost trade-off

M. Lim, D. Braha, S. Wijesinghe, S. Tucker and Y. Bar-Yam

EPL, **79** (2007) 58005

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

September 2007

EPL, **79** (2007) 58005
doi: 10.1209/0295-5075/79/58005

www.epljournal.org

Preferential detachment in broadcast signaling networks: Connectivity and cost trade-off

M. LIM[1,2(a)], D. BRAHA[1,3], S. WIJESINGHE[1], S. TUCKER[4] and Y. BAR-YAM[1]

[1] *New England Complex Systems Institute - Cambridge, MA 02138, USA*
[2] *Brandeis University - Waltham, MA 02454, USA*
[3] *University of Massachusetts Dartmouth - North Dartmouth, MA 02747, USA*
[4] *Sandia National Laboratories - Albuquerque, NM 87123, USA*

received 10 May 2007; accepted in final form 17 July 2007
published online 6 August 2007

PACS 89.75.Fb – Structures and organization in complex systems
PACS 89.75.Hc – Networks and genealogical trees
PACS 84.40.Ua – Telecommunications: signal transmission and processing; communication satellites

Abstract – We consider a network of nodes distributed in physical space without physical links communicating through message broadcasting over specified distances. Typically, communication using smaller distances is desirable due to savings in energy or other resources. We introduce a network formation mechanism to enable reducing the distances while retaining connectivity. Nodes, which initially transmit signals at a prespecified maximum distance, subject links to preferential detachment by autonomously decreasing their transmission radii while satisfying conditions of zero communication loss and fixed maximum node-hopping distance for signaling. Applied to networks with various spatial topologies, we find cost reductions as high as 90% over networks that are restricted to have all nodes with equal transmission distance.

Introduction. – The understanding of complex networks with physical links has been improving [1,2] yielding insights into network robustness to failure and attack [2–6], navigability [7], jamming and congestion [8–13], speed of information propagation [14], and efficient routing [15–19]. In this paper we study networks without physical links connected through virtual links created when a node sends out a broadcast signal by message carrier (*e.g.* radio wave, chemical) to all nearby nodes. Among a large number of such biological networks, neuronal molecular signaling (*e.g.* paracrine signaling) often involves the secretion of chemical signals onto a group of nearby target cells [20]. Broadcasting through electromagnetic transmission is used in multihop wireless networks, where messages may traverse multiple wireless links [21]. In both cases, localized broadcast transmission imposes key considerations. First, all nodes within the broadcast range of a transmitting node receive the signal, and only these nodes receive the signal (probabilistic distance-dependent links can occur [22]). Second, cyclical retransmission of a signal is avoided using refractory periods or other methods. Third, there is a signal lifetime or a maximum number of retransmissions so as to discard stale information and prevent system overload. Fourth, to enable a node to respond to distinct signals from multiple other nodes smart protocols [23] use conflict avoidance, while many biological networks, including neural networks, use a large number of distinct chemicals for multiple channels [24,25], and radio networks often use labeled signal packets. Finally, such networks expend resources to transmit signals, with a cost that increases with the distance of transmission, a key consideration in the design of networks, whether chemical or radio.

When barriers and other physical constraints effectively fix the spatial location of nodes and connectivity (whether direct or mediated) is required for all node pairs, the transmission distance of each node is the only variable of the system. Since the cost in energy or chemical messenger to establish links grows as a power of the transmission distance, D^{α}, where $2 \leqslant \alpha \leqslant 4$ for electromagnetic waves as well as chemical signals, the best cost reduction scheme will come from an effective reduction of the transmission distance. Truly autonomous wireless nodes

(a) On leave from the National Institute of Physics, University of the Philippines.

are self-powered, typically by batteries. Biological cells have limited stores of available energy and both energy and material considerations are key concerns. In either case reducing the distance of transmission to reduce costs is important.

In the following we consider a network of point nodes transmitting and receiving signals. This may be considered a model of multihop wireless networks [26–30], or of biological cells with chemical signaling. The underlying assumption of much of the earlier work is that nodes are randomly distributed [31] or node location can be controlled [21]. In many systems, however, node location is constrained, *e.g.* by the means by which nodes are distributed. As such, widely known methods of preferential attachment [2] and link rewiring [32] are not applicable. Instead, we propose a simple "preferential detachment" algorithm that minimizes the number of links in high-density areas while maintaining links in low-density areas.

Unlike continuum percolation [33] and other local information-based generative network models [34], which emphasize growth by establishing links, our method stresses the role of pruning. Synaptic overgrowth and pruning [35] is a standard process for changing networks during the maturation of the mammalian neural system [36]. We evaluate the benefits of our preferential detachment algorithm for network formation in comparison with uniform transmission distance networks for various node topologies. We find that preferential detachment results in significant cost reduction, saving 80–90% for a wide variety of network topologies. It also results in the reduction of signal congestion. Thus, our work, while distinct in its approach, can also be considered as a congestion reduction process (*i.e.* "congestion aware" protocol [1,6,11,17–19]).

Network model. – We distribute N nodes in a square $s \times s$ grid where the i-th node broadcasts a distance D_i from its location, (x_i, y_i). If another node j is at a distance $d_{ij} \leqslant D_i$, a link is established from i to j with a path length $L_{ij} = 1$ from i to j. Since D_i is not necessarily equal to D_j a link from i to j does not imply a link from j to i. Upon receiving a signal, a node retransmits the signal up to a maximum of h links. We assume functionally identical nodes that control only their own transmission power. We perform simulations on networks having different topologies all with $N = 256$ and $s = 600$ and then characterize the scaling behavior of the network properties to larger network sizes.

We characterize each network using: 1) the reachable pairs fraction $R = n/N(N-1)$, where n is the number of distinct ordered pairs (i, j) such that i can transmit to j through less than h links; 2) the average path length $L = \Sigma L_{ij}/n$ of all reachable pairs; and 3) a normalized network cost $C = (\Sigma D_i^2)/D_0^2$, where D_0^2 is the cost for a node at the center of the grid to broadcast over the entire area (D_0 is half the grid diagonal length $= s\sqrt{2}/2$ and $L_{0j} = 1$ for all j) where the scaling exponent of cost with distance is $\alpha = 2$. We use $\alpha = 2$ as the reference case, which is the power needed in free space, or the amount of chemical signal needed with a chemical wave propagation front. α increases for other assumptions of power or chemical spreading or dissipation with distance. Our analysis of the benefits of reduction of D_i is then conservative as the higher values, $\alpha > 2$, yield greater cost reductions.

Uniform distance networks. – Uniform transmission distance networks, where $D_i = D$ for all nodes i, have previously been studied as a percolation problem in a two-dimensional random lattice in which bonds are determined by the distance between sites [37]. N sites are randomly placed in an $s \times s$ square grid. When the distance between two sites is less than r_s, a bond is formed between them. For r_s greater than the critical (percolation) radius $r_\mathrm{s} = r_\mathrm{sp}$, one can find a series of links that traverses the space in each linear dimension. Monte Carlo simulations have shown that $r_\mathrm{sp} = (1.06 \pm 0.03)2s/\sqrt{(\pi N)}$ [37]. The analytic solution for N nodes in a disc of unit area is given by $\pi r_\mathrm{sp}^2 = (\log(N) + c(N))/N$, which yields a fully connected network for $c(N) \to \infty$ [38]. We can use these results to infer properties of our network for random distributions of sites. For our uniform distance network (fig. 1b, $h = 20$ hops), we expect that the average path length L reaches its peak value, $L(r_\mathrm{p} \approx 50) \approx 10.5$ hops, near the critical radius of percolation. The critical radius should also be the value of most rapid change of the reachable pairs fraction R. If we do not limit the maximum number of hops h (as we do in subsequent studies), r_sp gives a lower bound on the node transmission radius for boundary-to-boundary connectivity, albeit in the presence of "dead spots" indicating isolated clusters ($R < 1$). Imposing substantially smaller h necessarily increases all limiting radii values, thus r_sp is a strict lower bound. For the node distribution in fig. 1a, we calculate $r_\mathrm{sp} = 45 \pm 1.3$, which is consistent with fig. 1b. While percolation is concerned with widespread communication, our concern is complete communication over a fixed set of nodes.

Figure 1a shows the uniform distance network with minimum radius that still is fully connected, and thus is at minimum cost, formed by $N = 256$ nodes distributed randomly over a 600×600 space with $D_i = D = 60$ (for all nodes i), $h = 20$ hops, and $C = ND^2/D_0^2 = 5.12$. Two concerns arise with uniform distance networks: 1) The existence of the D-determining proverbial "weak link" —network fragmentation results when any node is unreachable; and 2) Unnecessary transmission cost in regions of high node densities, *i.e.* $D \gg d_{ij}$. The first concern can be addressed by increasing the radius of transmission to be larger than the minimum distance. The solution to the latter is selectively reducing D_i in high-node-density locations.

An apparently intuitive solution, using local density information for selectively reducing D_i, can be shown to be generally ineffective. Savings in regions of high density

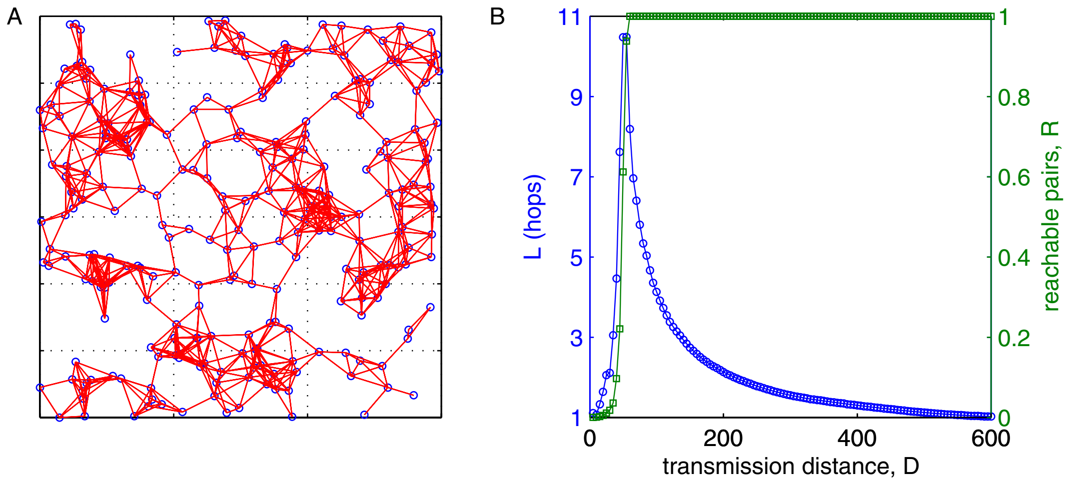

Fig. 1: (a) Uniform distance network of 256 nodes (circles) in a 600×600 grid with transmission distance $D = 60$. Links are shown where inter-node distance is one hop ($d_{ij} = 1$). (b) Variation of the average path length L (circles) and reachable pairs R (squares) for the network in (a), $h = 20$ hops.

are often compensated for by unnecessary costs outside that region when a single functional form is used to adjust transmission for all nodes. This can be seen through direct analysis of a three-node network using, *e.g.* $D_i = D_\mathrm{M}/\rho_\mathrm{local}$, where D_M takes the value of the maximum transmission distance [39]. More generally, setting the radius based upon local density is ineffective because the density is not isotropic, so that nodes that are near the edges of clusters defeat optimization by simple algorithms. We are thus motivated to find an adaptive method that overcomes this limitation.

Preferential detachment. – In our preferential detachment algorithm, we consider adaptive adjustment of transmission, reducing D_i until a minimum criterion for effective transmission is breached. In addition to complete network connectivity, we further restrict the networks to a maximum number of allowed hops h. The adaptive process begins by setting all transmission distances to a large value and performing synchronous reduction. When network connectivity breaks ($R < 1$), we incrementally increase the radius so that the network is fully connected. The adjustment of individual node radii then occurs as an asynchronous update of each node as follows:

1) Node i broadcasts a signal to all nodes. Nodes receiving the signal respond to the initial request with a response that can be detected by the original transmitter as a confirmation of receipt. Signal retransmission is allowed until h is reached.

2) Node i decrements its transmission distance by a fixed amount ($D'_i = D_i - \delta$).

3) Node i resends a signal.

4) If node i receives the same number of receipt replies, it returns to step 2). Otherwise, the node incrementally increases and fixes its transmission strength.

We note that the sender need only measure the aggregate magnitude of the response signal, and need not identify each of the signals separately. This enables a wide range of applications including chemical and wireless signals. The cycle repeats until all nodes have set their transmission distances (the number of such update cycles is bounded by D_M/δ, where D_M takes the value of the initial (maximum) transmission distance. The nature of the algorithm ensures that the overall normalized cost $C = \Sigma D_i^2/D_0^2$ is equal or better than that of the uniform distance network while ensuring that $L < h$. Moreover, it is guaranteed that no node can reduce its transmission distance without violating this condition.

We measure the efficacy of adaptive networks with respect to uniform distance networks using eight nodal topologies for 256 nodes chosen to represent a variety of geographical or nodal deployment constraints. In fig. 2, the coordinate origin (0, 0) is the bottom-left corner of each panel: A) random; B) random in three 200×200 clusters centered at coordinates (100, 100), (300, 400), (500, 200) with 50 nodes per cluster, and the remaining nodes randomly distributed over the 600×600 grid; C) 60% within a 200-radius central cluster with coordinates $(\rho\cos\theta, \rho\sin\theta)$, where ρ is randomly generated in the range (0, 200) and θ in the range of (0, 2π), 40% randomly distributed; D) star (five 200×200 randomly distributed clusters centered at x-y coordinates (100, 100), (100, 500), (500, 500), (500, 100), and (300, 300) with 50 nodes per cluster except the central cluster with 56 nodes); E) uniform lattice; F) radial distribution with coordinates $(k\cos[2\pi k(k+1)/96] + 300,\ k\sin[2\pi k(k+1)/96] + 300)$ and k is an integer from 1 to 256; G) distributed along preset lines; and H) random walk starting at the center.

Figure 3 compares the average path length (L) and cost (C) for the adaptive (circles) and uniform radii (square) networks for each node topology for varying h (5 to 30 hops

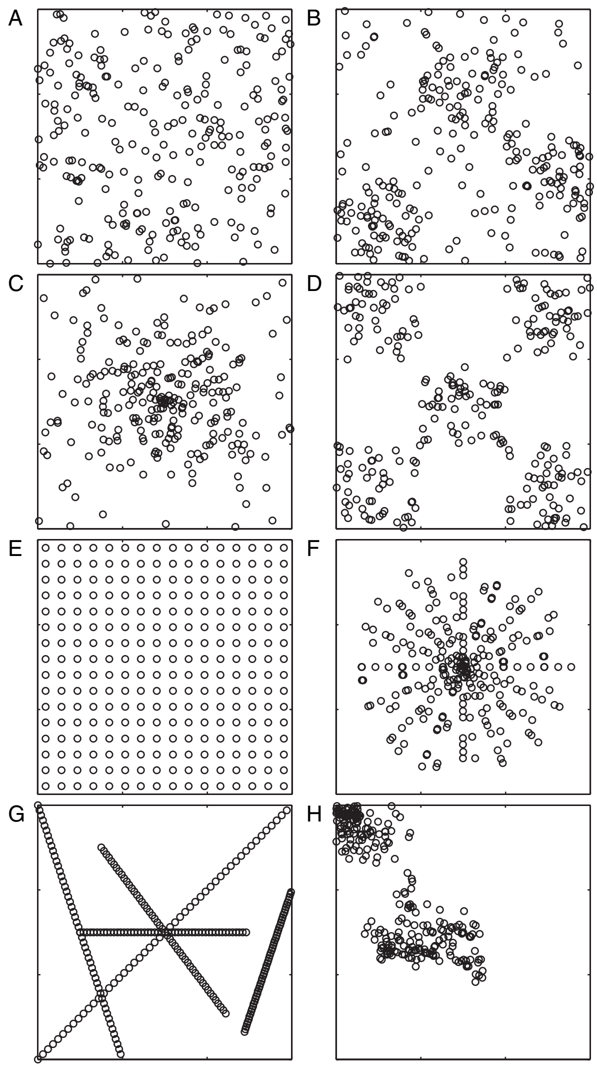

Fig. 2: Test nodal distributions: (a) random; (b) random in three clusters; (c) 60% within a 200-radius central cluster, 40% randomly distributed; (d) star configuration; (e) uniform lattice; (f) radial; (g) lines; and (h) random walk starting at the center.

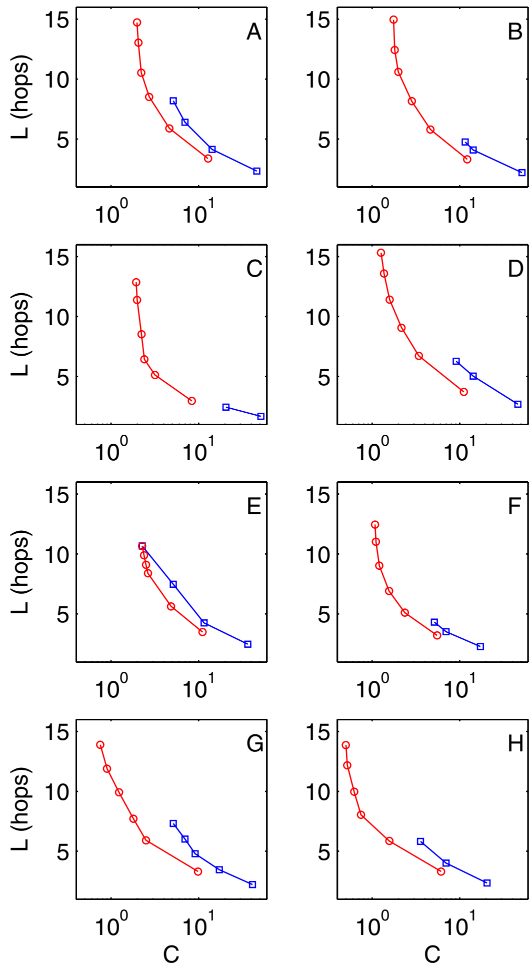

Fig. 3: Panels correspond to node distributions (fig. 2) for adaptive (circles) and uniform radii (squares) methods. Points indicate the minimum possible cost C and average path length L with $h = 5$, 10, 15, 20, 25, 30. In general, L increases with h.

in 5-hop increments). In general, the adaptive network provides significant cost savings given the same h over the uniform distance network. More specifically, we observe the following trends.

For larger values of h, the largest nearest-neighbor distance sets the minimum value of D. Figure 3c shows that for $h > 5$, the uniform radii method reached its limit due to the presence of a single relatively isolated node (fig. 2c, bottom center). In general, L increases with h.

The uniform distance and adaptive network solutions converge to the same value for large values of h if and only if a constant nearest-neighbor distance exists. In fig. 3e, convergence is achieved at $h = 30$ hops: the optimum distance is the minimum node-to-node distance and 30 hops exactly cover the distance from corner-to-corner in the uniform grid. It is worth noting that the line topology (fig. 2g) would have belonged to this category had all the lines been joined together. In practical situations, a break in the line may have been caused by a few nonfunctional nodes at critical junctions and highlights the strength of adaptive networks in allocating increased power output only at the boundary nodes.

In regions of high density, the adaptive method significantly reduces node transmission when subject to the h constraint. The significant cost savings can alternatively be used to reduce L for the same total cost (fig. 3).

We may take h to represent transmission lifetime and the number of signals that are being transmitted and received by a node increases with this lifetime. Taking h as a proxy for network load, we compare the performance of the adaptive and uniform distance networks using two metrics: a) relative cost $\kappa = C_{\mathrm{A}}/C_{\mathrm{U}}$; and b) length factor $\lambda = L_{\mathrm{A}}/L_{\mathrm{U}}$. Figure 4 shows the tradeoff between κ and λ for different topologies and h values. For this study we considered both $\alpha = 2$ and $\alpha = 4$. Topology-dependent effects are particularly evident for: a) fig. 2c topology,

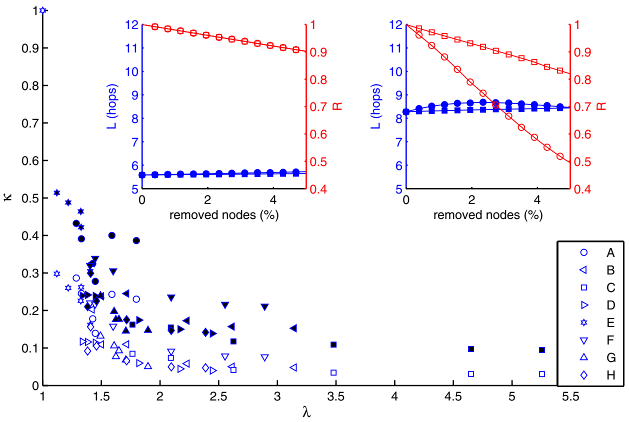

Fig. 4: Relative power consumption $\kappa = C_A/C_U$ and relative path length $\lambda = L_A/L_U$ of adaptive networks with respect to uniform distance networks ($\alpha = 2$ (filled symbols) and $\alpha = 4$ (open symbols)). Inset: network tolerance for uniform distance (left) and adaptive (right) networks against random failures (squares) and attacks (circles). Average path length L (blue on-line filled symbols) and reachable pairs ratio R (red on-line open symbols) show increased sensitivity of adaptive networks to attacks. Plots are averaged over 200 random test node distributions ($h = 15$).

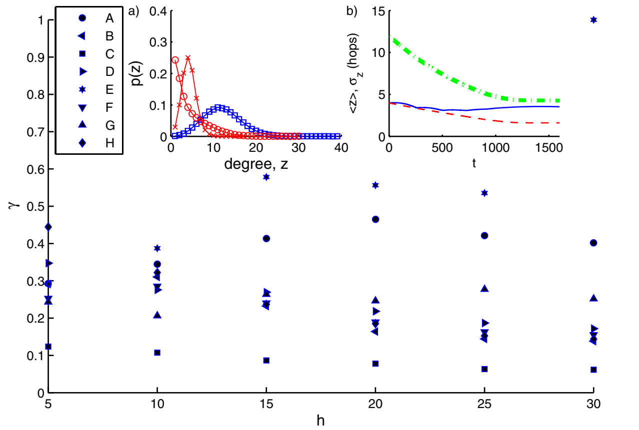

Fig. 5: The congestion at the node level drops with the use of an adaptive network as shown by the variation of the mean in-degree ratio $\gamma = I_A/I_U$ with the maximum packet lifetime, where I_A and I_U are the mean number of incoming links to a node for the adaptive and uniform distance networks, respectively. Inset: a) adaptive radius network in-degree (crosses) and out-degree (circles) distributions compared to the uniform radius network degree distribution (squares). (b) Degree distribution transition during preferential detachment. Mean of the degree distribution (dash-dotted line), and in-degree standard deviation (thin dashed line), monotonically decrease, the out-degree standard deviation (thin solid line) is non-monotonic, indicating a transition to a long-tailed distribution. Plots are averaged over 100 random test node distributions ($h = 15$).

where increasing h results in a marginal improvement of cost but results in a significant length factor change; and b) fig. 2e topology, where the uniform grid creates unique stepwise relationships between h and D. In all other cases where a smooth distribution of the nearest-neighbor distances exists, we obtain $\kappa \sim 0.2$ which translates to about 80% cost savings at the expense of a two- or three-fold length factor change under the same network load (h). In some cases the improvement exceeds 90%. Larger improvements are obtained when $\alpha = 4$ (fig. 4, open symbols).

Preferential detachment also results in a reduction in the mean values of the in- and out-degree distributions. While the shape of the in-degree distribution is almost unchanged, that of the out-degree distribution is positively skewed: very few nodes are left with a large number of outgoing links (fig. 5, inset). We show the time evolution of the degree distributions from the uniform radius network ($t = 0$) towards the final adaptive radius network. The mean and standard deviations of the in- and out-degree distributions for each iteration show positive skewing and stretching process for the out-degree distribution. We note that distinct in- and out-degree distributions have been found in large problem-solving networks with different constraints and asymmetries [40].

Adaptive networks are sensitive to attacks by removal of the highest-degree nodes (fig. 4, inset). After the removal of 5 nodes by random node failure or attack (2% of the total), the reachable fraction, R drops to 0.93 and 0.79 for random node failure and attacks, respectively. For the uniform distance network R decreases to 0.96 for both random failure and attack. The transmission distance, L increases by 0.1 hop (random failure) and 1.4 hops (attack) for the adaptive radius network. No significant change is observed for the uniform radius cases. We note that failure and attack properties are not designed for in this preferential detachment process. Modification of the methodology to enable different failure and attack properties is possible.

The reduction in the average number of incoming links has additional benefits. For a constant signal transmission rate, the number of incoming links (in-degree) [41] to a node is an indication of the congestion at the node. The relative mean in-degree of the adaptive to the uniform distance network γ (fig. 5) is significantly less than unity ($0.1 < \gamma < 0.6$ except when $h = 30$ for the uniform grid network). A drop in the number of incoming links is accompanied by a corresponding reduction in the likelihood of signal interference. A system using such a network needs fewer independent channels, *i.e.* fewer distinct chemicals or transmission frequencies. As mentioned in the introduction, our algorithm also reduces congestion [6,11,14–17,20]. Existing congestion reduction methods include message specific retransmission (*e.g.* listening and transmitting within a limited frequency range, responding only to specific chemical transmitters, not relaying messages from certain nodes); or by selectively inserting absorbing material (*e.g.* cellular matrix, absorbing walls) to inhibit transmission between specific nodes that are proximate.

The scaling (and scalability) of our results to larger system sizes can be analyzed from three perspectives: 1) the maximum number of hops needed to cross the system; 2) the characteristic power requirement on a node; and 3) the time it takes for the recipient to receive a message. For a network with a largely uniform density of nodes and ignoring corrections to scaling due to fluctuations, we can analyze the analogous system of N discs of radius r in a grid with side s, with a network density $\rho = N/s^2$. First, for fixed r the maximum number of hops (h) needed to cross the system increases linearly in each linear dimension, $h \sim s/r$; it therefore grows as the square root of the density or network size, *i.e.* $h \sim s\sqrt{\rho} = \sqrt{N}$. Second, since the characteristic cost of a node is proportional to its transmission area, $P = r^2$, the characteristic cost decreases with increasing density: $P \sim s^2 N = 1/\rho$. Finally, the time it takes for a recipient to receive a message depends on the messaging requirements of the network. If communication is global so that there is equal probability for any two nodes to communicate, the required time scales with the maximum number of hops and thus increases with linear dimension and the square root of density $t \sim h \sim s\sqrt{\rho}$. On the other hand, if it is more likely for local nodes to communicate, the time is independent of system size.

Conclusions. – We have shown that preferential detachment, adaptively reducing transmission distance, given fixed node locations and fixed signal lifetime, results in lower cost and efficient channel usage (lower number of incoming links). Specifically, it provides an average of 80% and as much as 90% transmission cost savings and a 40% to 90% drop in the average number of incoming links per node. While the maximum lifetime is constrained, the average lifetime (number of hops) of a signal increases by a factor of 2 to 3.

* * *

We thank R. Cohen for his comments. This work was supported in part by Sandia National Laboratories under US DoE Contract DE-AC04-94AL85000.

REFERENCES

[1] Boccaletti S. *et al.*, *Phys. Rep.*, **424** (2006) 175.
[2] Albert R. and Barabasi A. L., *Rev. Mod. Phys.*, **74** (2002) 47.
[3] Crucitti P. *et al.*, *Physica A*, **340** (2004) 388.
[4] Shargel B. *et al.*, *Phys. Rev. Lett.*, **90** (2003) 068701.
[5] Cohen R. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 4626.
[6] Huang L., Yang L. and Yang K., *Europhys. Lett.*, **72** (2005) 144.
[7] Kleinberg J. M., *Nature*, **406** (2000) 845.
[8] Toroczkai Z. and Bassler K. E., *Nature*, **428** (2004) 716.
[9] Gupte N. and Singh B. K., *Eur. Phys. J. B*, **50** (2006) 227.
[10] Wu J. J. *et al.*, *Europhys. Lett.*, **74** (2006) 560.
[11] Cholvi V., *Physica A*, **366** (2006) 571.
[12] Guimera R. *et al.*, *Phys. Rev. E*, **89** (2002) 248701.
[13] Echenique P., Gómez-Gardeñes J. and Moreno Y., *Europhys. Lett.*, **71** (2005) 325.
[14] Fortunato S., *Physica A*, **348** (2005) 683.
[15] Danila B. *et al.*, *Phys. Rev. E*, **74** (2006) 046114.
[16] Donetti L., Neri F. and Munoz M. A., *J. Stat. Mech.* (2006) P08007.
[17] Yan G. *et al.*, *Phys. Rev. E*, **73** (2006) 046108.
[18] Echenique P., Gómez-Gardeñes J. and Moreno Y., *Phys. Rev. E*, **70** (2004) 056105.
[19] Liu Z. H. *et al.*, *Physica A*, **370** (2006) 843.
[20] Purves D. *et al.*, *Neuroscience* (Sinnauer Associates, Inc., Sunderland, MA) 2001.
[21] Ramanathan R. and Rosales-Hain R., *IEEE INFOCOM 2000*, **2** (2000) 404.
[22] Medo M., *Physica A*, **360** (2006) 617.
[23] Arumugam R., Subraminian V. and Minai A. A., *Proceedings of the 4th International Conference on Complex Systems, Nashua, NH (2002)* (Springer, Heidelberg) 2007.
[24] Brezina V. and Weiss K. R., *Trends Neurosci.*, **20** (1997) 538.
[25] Laughlin S. B. and Sejnowski T. J., *Science*, **301** (2003) 1870.
[26] Goldsmith A. J. and Wicker S. B., *IEEE Wireless Commun.*, **9** (2002) 8.
[27] Lin Y. D. and Hsu Y. C., *IEEE INFOCOM 2000*, **3** (2000) 1273.
[28] Mikkonen J. *et al.*, *IEEE Commun. Mag.*, **36** (1998) 112.
[29] Iyengar S. S. and Brooks R. R., *J. Parallel Distributed Computing*, **64** (2004) 785.
[30] Wood A. D. and Stankovic J. A., *Computer*, **35** (2002) 48.
[31] Molisch A., *Wireless Communications* (Wiley-IEEE Press) 2005.
[32] Watts D. J. and Strogatz S. H., *Nature*, **393** (1998) 440.
[33] Glauche I. *et al.*, *Physica A*, **325** (2003) 577.
[34] White D. R. *et al.*, *Phys. Rev. E*, **73** (2006) 016119.
[35] Chechik G., Meilijson I. and Ruppin E., *Neural Comput.*, **10** (1998) 1759.
[36] Huttenlocher P. R. and De Courten C., *J. Neurosci.*, **6** (1987) 1.
[37] Pike G. E. and Seager C. H., *Phys. Rev. B*, **10** (1974) 1421.
[38] Gupta P. and Kumar P. R., in *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming*, edited by McEneaney W. M., Yin G. and Zhang Q. (Birkhauser, Boston) (1998).
[39] Lim M. *et al.*, unpublished.
[40] Braha D. and Bar-Yam Y., *Phys. Rev. E*, **69** (2004) 016113.
[41] Wasserman S. and Faust K., *Social Network Analysis: Method and Applications* (Cambridge University Press, Cambridge) 1994.