Pandemic Math: Stopping Outbreaks

Chen Shen and Yaneer Bar-Yam
New England Complex Systems Institute

Math to explain pandemics: Without including all the details, transmission is like matrix multiplication where each person can have the disease and be contagious or not.

Start with a list of everyone with either a "0" they are not sick or a "1" they are sick. Here there are two sick people.

$$V = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

The contact between people is a matrix that says who has enough contact with another person to infect them.

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \tag{2}$$

Since people only are in contact with a few other people out of all the people in the world this is a sparse matrix with mostly zeros. This is the contagion network.

Time is multiplication. Every day the matrix multiplies the vector to say what infections happen the next day. If day "0" is the day of the first infection from an animal, then day 1 is

$$AV \tag{3}$$

day 2 is

$$A^2V \tag{4}$$

the 10th day is

$$A^{10}V \tag{5}$$

and so on.

The multiplication can be summarized by a number, how connected the matrix is, called R_0. On the 10th day the number of infected people is

$$R_0^{10} \tag{6}$$

The most important question is whether it is greater than or less than 1. If it is greater than 1 it is a growing exponential. If it is less than 1 it is a shrinking exponential so that the outbreak will go away. Rather than days, we should count infectious periods because it takes a few days for an infected person to be able to infect someone else.

The matrix is not the same from day to day because people do different things. Researchers often think in terms of a typical matrix, not the specific contacts that are happening. What is most important is that we can choose to act to change the transmission network by how we behave.

If we reduce the number of sick individuals to zero, then the pathogen disappears and the matrix of connections doesn't matter. If it is not there it can't infect anyone. To get there we need to change the connectivity matrix for long enough for the number to be reduced to zero. This may sound hard, but this is an exponential process. Just as it can grow rapidly, it can also shrink rapidly.

If we were able to eliminate all of the connections, it would go to zero in one infectious period—which typically is a few weeks. This works no matter how many infected individuals we have to start with. That is the key to social intervention. It doesn't have to take a long time if it is very effective.

If the connections are not quite zero then it takes longer. If the outbreak is small it still takes only a few infectious periods. If it is large, the time to reduce the epidemic to zero is also longer, but it is exponential so it isn't very long. A few infectious periods makes a big difference.

There are four ways to change the transmission from a growing to a shrinking exponential:

The first is to isolate sick individuals, or people they touched. This targeted approach reduces connections to the specific people we know are contagious. If we know this exactly others can act as usual.

The second is to identify individuals who have symptoms and isolate them even if they don't have the specific disease. The earlier this is done the better.

The third is to generally reduce social connectivity. This happens by increasing social distance, wearing masks, washing hands, wearing gloves, not shaking hands, not having gatherings/parties, not touching what other people have just touched, not going into the same space with others. Which actions are most helpful depends on the way things are transmitted.

The fourth is to separate the matrix into blocks by stopping transportation or contact between groups. This makes the matrix into a block diagonal so individuals within a block can only infect or be infected by others in that block.

$$A = \left(\begin{array}{cccc|cccc|cccc|cccc} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right) \tag{7}$$

The shaded numbers that connect the smaller matrices on the diagonal are all zero.

There are also medical interventions. Improving care can make the disease less severe or deadly. A vaccination, if we have it, gives immunity.

The most important thing to realize is that we have choices that matter. Exponential growth can be turned into exponential decline if we choose to act.