A Mathematical Theory of Strong Emergence Using Multiscale Variety

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We argue conceptually and then demonstrate mathematically that it is possible to define a scientifically meaningful notion of strong emergence. A strong emergent property is a property of the system that cannot be found in the properties of the system's parts or in the interactions between the parts. The possibility of strong emergence follows from an ensemble perspective, which states that physical systems are only meaningful as ensembles rather than individual states. Emergent properties reside in the properties of the ensemble rather than of any individual state. A simple example is the case of a string of bits including a parity bit, i.e. the bits are constrained to have, e.g., an odd number of ON bits. This constraint is a property of the entire system that cannot be identified through any set of observations of the state of any or all subsystems of the system. It is a property that can only be found in observations of the state of the system as a whole. A collective constraint is a property of the system, however, the constraint is caused when the environment interacts with the system to select the allowable states. Although selection in this context does not necessarily correspond to biological evolution, it does suggest that evolutionary processes may lead to such emergent properties. A mathematical characterization of multiscale variety captures the implications of strong emergent properties on all subsystems of the system. Strong emergent properties result in oscillations of multiscale variety with negative values, a distinctive property. Examples of relevant applications in the case of social systems include various allocation, optimization, and functional requirements on the behavior of a system. Strongly emergent properties imply a global to local causality that is conceptually disturbing (but allowed!) in the context of conventional science, and is important to how we think about biological and social systems. *Complexity 9: 15–24, 2004*

Key Words: emergence; emergent properties; biological systems; social systems; complexity

1. INTRODUCTION TO EMERGENCE

Emergence is a widely discussed concept in the study of complex systems [1–5]. There are two distinct uses of the term. One is used in the form "emergent behavior" to characterize properties of a system that are in some way (possibly in a particular way) not captured by the properties of the parts. The second is a temporal version in which a new kind of system "emerges" at some historical time without in some way being captured in the previously existing systems. In this article we start by considering the first version, the emergence of behaviors, and conclude by discussing how the two types of emergence might interact.

An important distinction has been made between two forms of emergent behavior "weak" and "strong" [3, 6, 7].

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The former [1–9] describes the difficult to understand micro-to-macro relationship between microscopic parts and their interactions with each other, and their collective macroscopic behavior. The latter [10–12] describes properties that are unique to the collective—cannot be identified through any observations of the parts, and is counter to the conventional perspective that parts determine the behavior of the whole. In the case of strong emergence, it is possible that the properties of the whole determine the behavior of the parts. The central debate surrounding strong emergence is whether it can exist as a true property of real systems. This debate is tied to discussions of what the scientific method can and cannot understand, and specifically whether science can understand the human mind. Weak emergence is considered to be scientifically meaningful as it adheres to the scientific method framework, which describes systems in terms of their parts, whereas strong emergence is evoked almost exclusively to suggest that properties of the human mind, specifically "consciousness," may not be understood through science. Thus, both camps agree that the scientific approach is characterized by considering only weak emergence, whereas the question of whether strong emergence exists is tied to the ability of the scientific method to explain the human mind. Those who believe in the existence of strong emergence disparage the weak emergent scientific approach as reductionist, and those who do not believe in strong emergence reserve this term for a stronger form of reductionism that neglects relationships between the parts and weak emergent properties in describing a system's behavior.

In a sense, any dependency between components of a system leads to the description of the whole being different from the description of the components, because the description of the components does not capture that dependency. The discussion of such dependencies in a mathematical framework has been considered through constraint analysis [13–15] and reconstructability analysis [16–18]. These formalisms analyze the question: When can the component dependencies simplify so that a system can be represented in terms of its parts (defined in this context as small groups of interdependent components)? The formalisms do not guarantee such a decomposition, allowing for the case that a system cannot be described in terms of parts. The formalisms also consider how to identify a description of the whole when it is composed of a particular description of its parts. Although these treatments relate to the concept of emergence, they do not directly relate to the concept of strong emergence. Dependencies, such as correlations, can arise from the interactions of components and in such a case can be inferred from the properties of the components and their interactions. Such dependencies are included in the concept of weak emergence.

In this article we will focus on and characterize a distinctive way that systems *cannot* be described in terms of parts and through this demonstrate scientifically meaningful definitions of strong emergence. This work builds on the development of a general expression for the representation of k-fold dependencies in a system (the multiscale variety) that was obtained in a previous article [19]. Using the multiscale variety, we will analyze systems for which dependencies exist between many variables, but for which subsets of the variables do not have the analogous dependency. The multiscale variety reveals anomalous behavior for such cases, and it captures many properties associated with strong emergence. The concept of strong emergence we identify appears to be directly relevant to the study of complex systems. We argue that there are cases in which strong emergent behavior can be identified in simple physical systems, but that it is especially of interest for biological and social systems.

The primary subject of the article, contained in Section 3, is the discussion of one form of strong emergence that is found in the properties of the system ensemble rather than the properties of a single microstate. A specific example of this type of strong emergence is described, a general mathematical theory describing strong emergence based on multiscale variety is presented, and various examples particularly in social systems are identified. Before discussing this type of strong emergence in detail, we develop, in Section 2, a wider typology of emergence that includes weak emergence as the first type, and two types of strong emergence as types 2 and 3. Type 2 is the one found in the properties of the system ensemble, whereas type 3 is found in the relationship between the properties of the system to those of the environment. We also include a "zeroth" type that considers the properties of a system that can be understood from the properties of the parts without any relationship between them. Finally, in Section 4, we briefly introduce the possible connection between type 2 emergence and evolutionary processes and the possibility of characterizing the differences in functional capabilities of evolved and conventionally engineered systems.

2. CONCEPTS OF EMERGENCE

The study of emergence is concerned with both physical properties and observations of systems. From an objectivist perspective, it is about how physical properties affect observations. However, because our understanding of systems arises from observations, it is also about how we identify or describe system properties from observations. The central question is: How are disparate observations of a system related? In order to make sense of discussions of observations and hence emergence, it is important to understand the concepts of scope and resolution (scale). We also have to be careful to recognize which observed properties are properties of the system and which are properties of the parts.

When we discuss scopes, we are referring to the observation of various parts/subsystems of the system as well as the system as a whole, whereas when we discuss scale, we always consider the system as a whole but at differing levels of detail. When considering the issue of scope it is important to also identify whether a subsystem is observed *in situ* or isolated from the rest of the system under observation. Strictly speaking, isolation may not be a well-defined process because all systems have environments; however, this is often ignored (only sometimes with justification, leading to the possibility that the attributions of properties are misidentified).

We often think of scale and scope as coupled because of the most common ways we encounter them. For example: consider observing a system through a camera that has a zoom lens. For a fixed aperture camera, the use of a zoom couples scope and resolution in the image it provides. As we zoom in on the image we see a smaller part of the world at a progressively greater resolution. This leads to a particular relationship of observations of parts and wholes, suggesting that when observing details of the system, the whole is not being observed. In order to discuss emergence effectively we must allow a decoupling of scope and resolution, so that the system as a whole can be considered at differing resolutions as well as part by part. For this purpose scale can be considered as related to the focus of a camera—a blurry image is a larger scale image—whereas scope is related to the aperture size and choice of direction of observation.

When studying emergence, it is important to recognize that properties that we often associate with a part are actually relational properties and therefore are properties of the system rather than of the part. For example, we often consider a particle's position to be a property of the particle. Although it is not essential for the central topic of this article, we note, however, that for an observer looking at an atom in isolation, the position of the atom has no meaning because the observer can choose any point in space as a reference frame. Such an observer centric reference frame would assign an arbitrary position to the particle. Thus, the particle position is a relative quantity not an absolute one. When we consider the intrinsic properties of a particle, its position is not relevant.

On the other hand, when a particle is considered as part of a system, relative locations of particles have meaning. However, it is often convenient to think about positions as being absolute by fixing the reference frame of the observer. In particular, for a system with many particles, there is one three-dimensional coordinate (6 dimensions with orientations, 12 with velocity and rotational velocity) that establishes a frame of reference for the system. Once this frame is given, all $3N^2$ relative locations of N particles can be specified from $3N$ position coordinates with respect to the frame of reference ($6N^2$ relative positions and velocities can be specified from $6N$ "absolute" positions and velocities). For this reason, it is convenient to think of particle positions as being absolute when part of an overall system. However, this convenience makes it difficult to realize that positions are relational (systems) properties rather than properties of the particles themselves.

FIGURE 1

Type A: Emergent behavior (Micro to macro)
- Type 0: Parts in isolation without positions to whole
- Type 1: Parts with positions to whole (weak emergence)
- Type 2: Ensemble with collective constraint (strong emergence)
- Type 3: System to environment relational property (strong emergence)

Type B: Dynamic emergence of new types of systems "new emergent forms."

Types of emergence.

With these concepts in mind (scale, scope, and care in assigning the properties of parts and systems), we can now discuss several concepts of emergence (Figure 1). We start with a very basic concept of emergence, which we call the zeroth form.

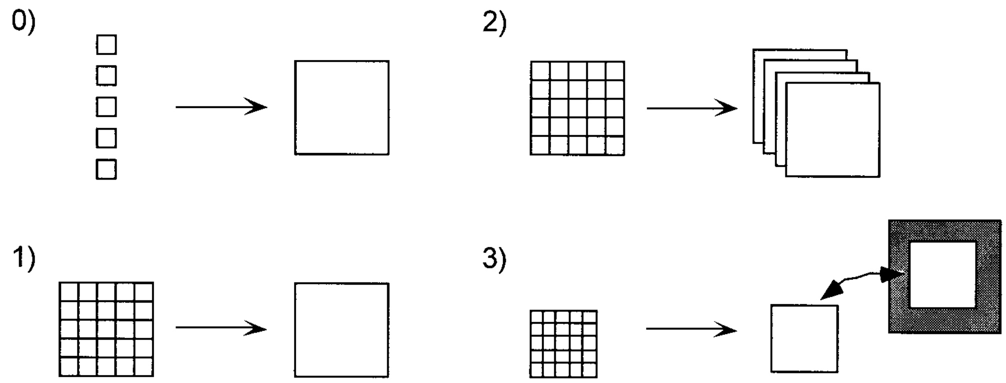
2.1. Type 0 Emergence

The zeroth form of emergence considers the properties of the whole system compared to the properties of the parts observed in isolation (Figure 2). When we think of a part "in isolation" such as an atom of oxygen, we identify its properties as that of any atom of oxygen. This information does not include where a particular atom of oxygen is located. It is then clear that if we know the properties of each atom, and the number of atoms of each type that form a particular system, in general we will not be able to infer many of the properties of the system. There are many systems that could be formed from the same atoms, with widely differing properties depending on how they are arranged. It is important to emphasize that this notion of emergence is not trivial in the context of the culture of science. After all, grand unified theories in physics claim to understand the universe without allowing for organization. Also, for many years the perspective of molecular biology has been that if two systems have the same (or even similar) sets of molecules, then they are for all intents and purposes the same. Clearly, when considering the culture of science, the issue of zeroth emergence as an important and nontrivial example of how collective behaviors are not contained in the behaviors of the parts is still an important one. However, we will not address this issue any further in this article.

2.2. Type 1 (Weak) Emergence

To introduce the first type of emergence, weak emergence, we consider the relationship of microscopic and macroscopic views of a system that differ only in precision. The microscopic behavior of a system is defined in terms of the

FIGURE 2



Schematic illustration of the four types of emergent behavior (see Figure 1, A0–A3).

positions and momenta of and interactions between all the particles, and the macroscopic behavior is defined in terms of a set of collective behaviors that are observable at a macroscopic scale. [Among the system properties that can be considered collective behaviors are physical properties such as pressure, temperature, magnetism, density waves, other types of waves, phase transition properties, spatio-temporal patterns like those of clouds, or similar patterns in biological or social systems such as patterns on animal skins or traffic jams, and other structural, dynamic or response properties of systems may also be included, e.g., the permeability of a wall made of bricks (We do not assume that these properties can all be described using the concept of weak emergence. Some of them may require strong emergence discussed later.)].

In the conventional view, the positions and momenta of all the particles $\{x,p\}$ and their identities (electron, proton, neutron etc.) uniquely define the state of the system, which is viewed as sufficient to define the microscopic as well as macroscopic properties of the system. We note that according to accepted physical law, the positions and velocities of the particles of the system, and their fundamental identities are sufficient to determine their pairwise interactions. This is a remarkable statement that is not at all obvious but has withstood the scrutiny of experiments. However, it is only true *when all particles are specified by positions and velocities, and not when they are in the presence of responsive media*, and therefore it is not true about a system defined generally. For example, a system consisting of a set of impurities that are embedded in a solid does not have this property. The interactions between the impurities do not follow just from their coordinates. Still, the concept of weak emergence allows one to specify not only the positions and velocities of all the particles but also the interactions between the particles. According to the general understanding of weak emergence, this information is sufficient to describe the system completely. Also according to this view, however, extracting the collective behavior from the behavior or the parts is difficult. Because of this difficulty in extracting the collective behavior, the concept of "emergence," which suggests some degree of mystery should apply.

More specifically, according to the standard view of weak emergence, collective behaviors of the system can not be readily recognized because it is difficult to extract them from the large amount of information present in the fine scale microscopic view. "Emergence" is the name given to this process because given the list of all the positions of all the particles, it is assumed that a computational and filtering process of "data mining" that would extract the collective behavior of the system would be extremely difficult, if not practically impossible, and therefore the notion of "emergence," should apply. However, this perspective suggests that the problem of observing collective behavior is practical as opposed to fundamental. Although it may be quite difficult to extract the large-scale view from the highly detailed fine scale information, the conventional and natural assumption is that it is possible in principle. This is just as a picture with a resolution of 1800×2400 pixels can be reduced to a picture of 300×500 pixels by local averaging. The picture with finer detail contains the information in a coarser picture. The field of statistical physics can be understood as an effort to obtain the macroscopic properties of systems from their microscopic properties. The successes of describing equilibrium systems, including phase diagrams and thermodynamic transitions from statistical averages over microscopic representations, demonstrates the validity of the approach of this field. Extensions to nonequilibrium systems exist in many contexts and provide support for the perspective that such treatments can be generalized to account for many phenomena. Below, we make a subtle distinction between different ways of approaching the microscopic to macroscopic relationship that allows us to show that weak emergence is insufficient to describe all collective behaviors, though as we will see, it is not an indictment of the statistical treatment of systems, but rather refines our understanding of such descriptions.

2.3. Type 2 (Strong) Emergence

The main purpose of this article is to introduce and describe the properties of type 2 strong emergence. We will discuss it at greater length in Section 3 below. Here we only provide a brief overview. Type 2 strong emergence arises from considering the set of possible states (ensemble) the system can be in rather than a particular state. Once the importance of an ensemble is recognized, there is an additional difference between observations of system components and observations of the system. The reason is that the component ensembles do not directly aggregate to the system ensemble. This was not the case for the states of the system, where the entire state of the system could be defined as the aggregate of the states of the components. The reason that the ensemble of the system is not just the ensemble of each part is because of the interdependence of the parts. Specifically the state of one part may determine (or be coupled to) the state of other parts. This, however, is not observed if we only look at each part separately. Although the statement that an ensemble of the entire system does not decompose into the ensemble of its parts is true in many cases, e.g., correlations between any two subsystems, we will focus on particular instances that are of direct importance to the question of strong emergence. When a system is faced with global constraints, the properties of an entire system may determine the properties of a part, without the properties of a part determining the properties of the whole system. This is consistent with the notion of strong emergence as it has been discussed historically. There is no strong emergence when the system is defined by constraints that act on each component. Only when constraints are defined that act on collectives and not on components does strong emergence occur. We will also show that there is a mathematical distinction between cases where there are such global constraints, supporting the importance of making a distinction between weak and strong emergence.

2.4. Type 3 (Strong) Emergence

Finally, we note that there is a type of emergence (which we call type 3 emergence), which also deviates from the common notion that behaviors of systems are contained in the properties of its component parts. An example of type three emergence is a lock and key. The properties of a key in opening a door are not contained in a description of the parts of the key. Instead they are contained in the relationship between the components of the key and the components of the lock. This relationship is not present in the description of the parts of the key by themselves. We can note that when viewing a system that includes both the key and lock, their relationship is that of a constraint that is not contained in the description of the parts themselves but rather in the description of the relationships between them. Still, in this case the ability of the key to open the door for a particular instance can be inferred from the structure of the parts themselves without reference to the ensemble of possible keys and doors.

When we consider that even at the molecular level, the behavior of proteins is often considered quite similar to that of a lock and key, with proteins fitting into one another, and enzymatic processes controlled by geometric fitting and chemical binding, the idea of type 3 emergence as a relationship between the system and aspects of the environment is clearly central to the function of complex systems in the world around us. Such cases are not contained in the descriptions of the parts in isolation, even if their properties can be defined, the relevance of these properties depends on the existence of the complementary molecules and substrates in the environment. Thus, even fully described, such relationships are not captured unless information about the environment is included. This is not contained in the conventional discussion of properties of a system as determined by the system itself.

3. THE ENSEMBLE PERSPECTIVE AND STRONG EMERGENCE

3.1. Ensemble Perspective

The perspective that a system can be defined as a unique microstate does not account for the recognition that any system we define has a multiplicity of microstates and multiple observations of the system cannot be performed on the same microstate but rather on the ensemble of states. Therefore, a property of the state of the system obtained by observations is actually an average over the ensemble. The ensemble is specified by $P(s)$, the probability of a particular state s of the system. If the value of the observed quantity is in state s is $A(s)$ then the observed quantity averaged over the ensemble is given by $\langle P(s)A(s)\rangle$. The description of the system and its properties is related to the state of the system s and to the ensemble $P(s)$ (We note that we are distinguishing between observations of a state and observations of forced transitions of the system. As we will show below, observations of the transitions of parts of the system can reveal the presence of global constraints, even when observation of the state of a part over time cannot. This is the reason that properties of the collective influence properties of the part, in transition, but not in state. Observations of transitions are called "off-diagonal" transitions in quantum mechanics.)

When we consider the description of the state of the system to be a statement of the state of all the parts (e.g., the position of all the particles) and the description of each part *in situ* to be a statement of its state (e.g., the position of each particle), the description of the state of the system is a compilation of the descriptions of the parts. The ensemble of a compilation of the description of the parts is the product space of the ensemble of the parts. In general this is different from the ensemble of the system itself. This difference can arise because of the interactions between the parts or because the system ensemble itself is affected by its environment. In either case we have an ensemble of the entire system $P(\mathbf{s})$, $\mathbf{s} = \{s_i\}$, $i = 1, \ldots N$, which is not the product of the ensembles of the parts $\bar{P}(\mathbf{s}) = \Pi_i \, p_{s_i}(s_i)$. The former includes interdependence, whereas the latter does not. From the point of view of observation, a more careful consideration of physics principles suggests that a single state does not correspond to a physically observable system and therefore is not a physically meaningful concept. Physics is only concerned with reproducible experiments and therefore with experiments that are performed on a system that is defined by a preparation process rather than a unique microstate.

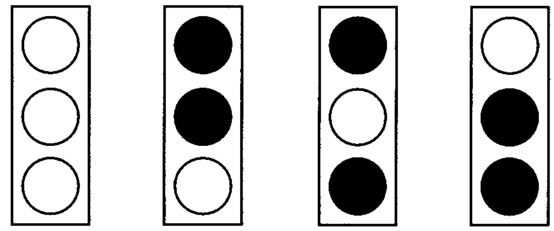
For quantum theory, the analog of considering the ensemble of the system is considering the density matrix. The shift from particle positions and momenta to an ensemble corresponds to a shift from the wavefunction to a density matrix and the response of a system is determined by its density matrix, not a particular wavefunction. In this article, we are not concerned with quantum issues, but rather with the semi-classical perspective that is generally considered sufficient to represent macroscopic behavior of systems (when collective quantum phenomena like superfluidity are not relevant because temperatures are not ultralow) even if they have complex structures.

3.2. Type 2 Strong Emergence

We focus on the properties of ensembles in order to explain the possibility of strong emergence. System ensemble properties may not be observable in the states of the components or the ensembles of the components. Specifically, we construct a system with a constraint on the whole that does not apply to any subsystem. Assume that we have a system with N bits. These bits are constrained to have an odd number of ON bits. An example of a three bit system is illustrated in Figure 3. If we look at any subsystem of the bits, the subsystem does not have the same constraint. Indeed, any possible arrangement of a subsystem is equally likely. Still, the constraint on the whole system is a property of the system. We see that this property cannot be inferred from observations of the components themselves, but rather only by examining the system as a whole.

The key to this observation is the recognition that properties of a system may be constrained in multiple relationships. Unlike conventional physical interactions that are only pairwise, there may be ensemble constraints that do not simplify to pairwise constraints. Pairwise interactions may also lead to multiparticle dependencies. However, multicomponent dependencies can arise from environmental effects that are not captured by pairwise interactions. In particular there may be constraints that only apply to a macroscopic subsystem or the entirety of the system. Such constraints are only observable through observations of the whole and not through combinations of observations of the components compiled into observations of the entirety.

FIGURE 3



Parity bit ensemble for a three-bit system with two possibilities represented by circles colored black and white. The system allows four (out of the usual eight) possible states that can be identified as constrained to allow only an odd number of white circles. Each bit has 50% probability of each possibility, and each pair of bits has 25% of each of the four possible states for two bits. Thus observations of any proper subset cannot reveal the existence of the global constraint. This system can be constructed by allowing all possible arrangements of any two bits and requiring the third bit (the parity bit) to satisfy the constraint. All bits satisfy this property and the system states are symmetric with respect to bit exchange. Remarkably, this implies that the state of any bit is completely constrained by the global constraint applied to the whole system, even though no observation of a bit reveals this. These properties appear to satisfy the conceptual description of strong emergent properties, and they provide for a distinctive multiscale mathematical signature found in Figure 4.

Moreover, it is interesting that any of the bits is totally constrained by the values of the other bits in the system. Given two bits, that can be set arbitrarily, the third bit must be specified so that the sum over the bits is odd. On the other hand, by looking at that bit or any subset of bits, one cannot see this constraint. The reason is that when considered over the ensemble of all possible states of the system, there is no net impact on the ensemble of the individual bit. Does this mean that it is impacted or not? The value of the individual bit is impacted by the values of the rest of the bits as far as a single state is concerned but not as far as an ensemble is concerned. This is the opposite of what one would say about the entire system, which is impacted in the ensemble picture but not in the state picture.

3.3. Mathematical Formulation

Previously [19], we have described a formalism that captures the multiscale variety of a system. This formalism considers the constraints that exist in a system and lead to collective behaviors. It separates behaviors that correspond to actions of k-fold components to obtain the amount of information necessary to describe a system at a particular scale of resolution. We can use this formalism for parity bit systems, as well as other cases that might display such behaviors.

The probability distribution for a parity bit system is given by ($s_i = \pm 1$)

$$P(\mathbf{s}) = \delta\left(\mathrm{mod}_2\left(\sum (s_i + 1)/2\right), 1\right)/2^{n-1.} \tag{1}$$

For the case of three bits (including the parity bit), we can directly calculate the values of the variety $V(k)$ as a function of scale (the information needed to describe the system at that scale including behaviors at larger scales):

$$V(k) = (V(1),\ V(2),\ V(3)) = (2,\ 2,\ -1), \tag{2}$$

or the variety at each particular scale $D(k) = (0, 3, -1)$. The following discussion explains how we can obtain the value of $V(k)$ and understand its importance. The value of $V(1)$ is the total amount of information necessary to specify the state of the system. We can see from the definition of the parity bit system that this is one less than the total number of bits, because the last bit can be specified in terms of the values of the others, so $V(1) = 2$. The fact that $D(1) = 0$ arises from the observation that all information in the system is redundant, i.e., is shared with the rest of this system. Indeed, a parity bit system is used in computers to provide for error checking. Whenever there is an error that occurs from noise, the parity bit can be used to notice that the error exists. If the sum over the bits is found not to be odd, then there has been data corruption. Because any bit has the possibility of error correction, it is clear that all bits of the system have redundancy. The existence of redundancy implies that $D(1) = 0$, consistent with the numerical result. Because $V(2) = V(1) - D(1)$, i.e., the only difference between $V(2)$ and $V(1)$ is the variety at scale one, this also means that $V(2) = V(1)$. Because a sum rule [19] specifies that $\Sigma_k V(k) = N$, this is sufficient to give all the results for three bits.

More generally, to understand the mutual dependency of the variables we recognize that the system has one less bit of information than the number of variables; however, this bit of information is a collective behavior of all the variables: all three bits are involved. Any pair of variables is independent, but all three variables together are mutually dependent, leading to a reduction in the total amount of information necessary to describe a particular state of the system. The negative mutual information at the highest scale reflects this interdependence.

For parity bit systems with $N = 4$ and $N = 5$, we can obtain $V(k)$ and $D(k)$ from an expression for $D(k)$ given in Eq. 9 of Ref. 19. $D(k)$ is $(0, 6, -4, 1)$ and $(0, 10, -10, 5, -1)$, and $V(k)$ is $(3, 3, -3, 1)$ and $(4, 4, -6, 4, -1)$, respectively. For arbitrary N (Figure 4):

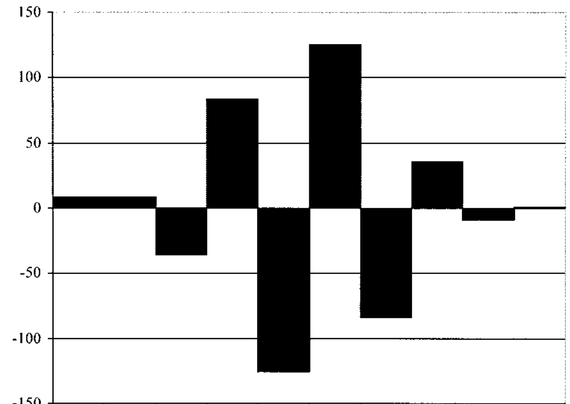
$$V(k) = N\delta_{k,1} + (-1)^k \binom{N-1}{k-1}$$

$$D(k) = N\delta_{k,1} + (-1)^k \binom{N}{k} \tag{3}$$

Oscillations occur as a function of scale. Such oscillations occur when there are constraints/dependencies on the values of multiple variables, without the analogous relationships between subsets of these variables. The oscillations are modulated by combinatorial factors arising from the number of possible subsets. These oscillations suggest that additional information is required to describe the system at the intermediate scales. Indeed, the information increases beyond the number of bits in the information of the entire system, potentially by a quite large amount. The reason for this is the existence of constraints that impact on each and every subsystem. Thus, in order to describe the behavior of a subsystem, additional information is needed.

The key to understanding the paradox that strong emergent quantities impact on the description of all subsystems

FIGURE 4



k	1	2	3	4	5	6	7	8	9	10
V(k)	9	9	-36	84	-126	126	-84	36	-9	1

Plot and table of the behavior of the scale-dependent variety $V(k)$ for a parity bit system with $N = 10$. The oscillations are a signature of the existence of a collective constraint impacting the information that is needed to describe the system at any scale of resolution. Note that the largest magnitude of the variety far exceeds the number of bits in the system. This occurs because as the resolution is increased, information from the large scale must be corrected at the next lower scale. Ultimately, the total amount of information is $N - 1$, the variety at $k = 1$. The negative varieties are counterintuitive but reasonably indicate the propagation of the constraint on the whole system through all subsystems.

is to realize that this is directly tied to the issue of resolution dependent observations. As the resolution is improved, the information available from large-scale observations is found to be in error and must be corrected in order to allow a description of the finer scale. This is important because we care about larger scale behaviors generally, and in this case the larger scale behaviors impact on the finer scale behavior even though the observation of the parts is not impacted by the observation of the collective.

We see that the scale dependent variety $V(k)$ shows quite generally the dependencies between sets of k variables. Dependencies between larger sets of variables that include these sets are also reflected in the value of $V(k)$. In Ref. 19 we argued that $V(k)$ represented the degree to which a system could respond at the scale k or larger. In what sense is a bit string with a parity bit able to respond at the level of the number of bits N? If we consider the effect of removing a single bit, the structure of the system is lost. If we consider the appearance of any subset less than N, it seems random. Still, there is a sense in which the system does respond at scale N, because it is constrained at the level of N bits to the states that it can occupy. This implies a nontrivial response of the system to environmental perturbations of order N bits. If the environment tries to flip all of the bits (if the total number of bits is odd), then the system will refuse. If the environment tries to flip them in some other coordinated way, the response of the system will be predicated on whether or not the change retains the total parity of the system. Thus at any intermediate scale, the system also responds either by allowing or disallowing a transition that is imposed from the outside. Transitions of any subset of bits will be allowed if the number of bits is even, but not if the number of bits is odd. This implies that the system has collective behaviors at many intermediate scales.

From the point of view of the environment, the different possible responses, either to change or not to change, have reciprocal impact on the environment. These effects imply that the environment is affected at the scale of N (i.e., actions of the system can impact on N coherently behaving components of the environment). Moreover, there is a possible impact at all (odd) scales. Because the environment may couple in various ways to the system, we can say that the environment will be impacted at all intermediate scales in a large variety of different ways. This is the meaning of the high magnitude of variety that occurs as a function of the scale of the system.

Naively, one might think that this phenomenon of oscillating scale-dependent variety and the existence of negative varieties is a flaw in the formalism. However, this is not the case because the formalism reflects a true phenomenon that characterizes the collective behavior of systems. This collective phenomenon is not just an abstract mathematical construct, but rather a real property that impacts how we interact with the world, in both biological and social contexts.

To understand the implications of such global constraints and the relevance of the mathematical formalism to the real world, consider a social organization that is constrained at the collective level but not at the level of any subset, for example, an organization that has a fixed and immutable total budget (or other allocated resource). This budget must be allocated among individuals and subgroups of the organization. However, given variations in the needs of the individuals and groups within the organization, that are subject to varying environmental demands, the allocation cannot be uniform across the system. It is easy to recognize that this allocation problem is quite difficult to achieve and requires extensive coordination in order for the constraint to be satisfied. Indeed, we see that this constraint has implications for the behavior of any subgroup of the system inherited from the constraint on the entire system. If a subgroup would like to increase its budget (say in response to a change in external demands), it must coordinate with another individual or group that will lower its budget. From the point of view of coordination, this problem is very difficult to solve. This difficulty is reflected in the fluctuations of the multiscale complexity that describes the existence of subsystem constraints on the system, which carries information about the behavior of the system in part and in whole.

In order to treat this problem mathematically, we assume for simplicity that each individual is assigned either a unit decrement or a unit increment of the resource. Given a total sum of zero, the multiscale variety for four or six variables would be: $V(k) = (3, 3, -1, 0)$ and $(4, 4, -5, 3, -1, 0)$. Notice that although the four-variable case has one less bit of information like the case of a parity system, the six-variable case has two less bits of information, and the fluctuations are substantially smaller than in the parity system. This occurs because the total system constraint in this case also has analogous subsystem constraints. Specifically, the system as a whole as well as any subsystem cannot be flipped unless there are equal numbers of decrements and increments. The effect of the constraint on large subsystems can also be seen, because, for example, any subsystem larger than half of the system cannot be all of one type.

Consider now the possibility of defining the system of interest to contain only part of the original strong emergent system. From the point of view of observations of the state of this subsystem, there is no way that we can see the constraint that exists at the larger scale. Observations of this subsystem reveal that all possible states will occur. Still, if we look at the response of this system to its environment (behavior), we may find that its response is not that of an unconstrained system, indeed its responses may often be highly constrained. We may even find that these constraints appear and disappear in an unaccountable way because the environment may be affecting the subsystem that we are looking at the same time it is affecting the rest of the system. For a parity bit system, when the environment attempts to flip an even number of bits in total, including those within the system of interest (the subsystem) and those outside the system of interest, but part of the larger system that has the constraint, then the bits flip. On the other hand, when it tries to flip an odd number of bits, the bits do not flip because the global constraint does not allow them to do so. The results would be mysterious if we did not pay attention to the bits that are outside of the system of interest, but that are nevertheless coupled to the system of interest by the global constraint. From these observations we can identify a distinct kind of inverse emergence, which suggests that a subsystem cannot be understood directly unless it is understood in the context of the whole. Thus the idea that higher-level system organization is necessary for understanding the behavior of a subsystem is manifest in this example.

3.4. Examples of Type 2 (Strong) Emergence

This idea of strong emergence can be seen in a variety of physical, biological, and social systems. It holds when global constraints affect individual behaviors, without analogous individual behavior constraints. Some examples in social systems are as follows:

- Allocation constraints (total budget; discussed above),
- A fixed number of entities in a group (players on a team),
- Collective tradeoffs (multiplayer zero sum game),
- Pairing (marriage without polygamy or homosexuality),
- Complex optimizing systems—global optimum does not imply local optimum (traveling salesman problem, scheduling),
- Frustrated interacting systems (seating people around a table with preferred adjacent partners),
- Steady-state flows (high density traffic, supply chains),
- Quota filling (course selection for a degree, filling seats in an auditorium),
- Global matching (market price supply and demand relationship),
- Constraining populations (number of minorities allowed or required),
- Other allocation systems (Congressional representatives allocated to states),
- Existence requirement (someone has to take out the garbage).

Examples from simple physical systems that display collective constraints include:

- Systems with boundary conditions leading to harmonic vibrations (periodic conditions or fixed boundaries),
- Frustrated interactions (antiferomagnetic systems),
- Soliton carrying systems,
- Steady-state flows,
- Systems at temperatures just below a phase transition (the local behavior includes fluctuations that do not reflect the ordering at scales above the correlation length; S. Gheorghiu-Svirschevski and Y. Bar-Yam, unpublished results).

These lists of examples suggest that strong emergent behavior is of general interest in the study of complex systems and is not restricted to a particular disciplinary context.

4. EVOLUTION AND THE EMERGENCE OF SYSTEMS

A final topic that we want to briefly discuss is the interplay between emergence of collective behavior and emergence of new forms of systems over time, particularly through evolutionary processes. In particular, we want to relate evolutionary processes to the concept of strong emergence that we have been discussing. The main point is simply the observation that selection is fundamental to both type 2 strong emergence and to evolution. Specifically, the generation of global constraints would seem to naturally arise from evolutionary processes. Thus, we would expect living organisms to demonstrate such strong emergent behaviors.

We can contrast systems that are formed through evolution with systems that are considered to be constructed. Machines are generally designed (though not necessarily so) in a way that enables bottom up planning in which parts can be composed together to achieve behaviors of the whole that are describable in terms of the behaviors of the parts. This is consistent with the concept of weak emergence, but not with the concept of strong emergence. There are important exceptions. One exception is the parity bit system discussed earlier, which is used in the context of computers. However, it is used not as part of the intrinsic functional capacity of the system, but rather as a means of detecting noise that is nominally outside of the operations of the system. Other examples include feedback control systems that impose constraints, optimization or goal seeking behavior on the system. These examples are particularly interesting in that they are traditional examples of the incorporation of systems thinking into engineering. A possible direction of future inquiry is studying the limitations conventional engineering places on system capabilities; some capabilities that might be exclusive to systems that have strong emergent behaviors. Even at a qualitative level, it appears intuitive that the lack of robustness of engineered systems to various perturbations is consistent with a lack of type 2 emergent properties. For example, the stability of a system suggests a constraint on the collective properties of a system that parts need not obey if they are "perturbed" from their original states. Given that robustness suggests an ongoing retention of a collective property in the context of many possible perturbations, the association of robustness to type 2 emergence seems clear.

SUMMARY

We have presented a characterization of properties of a system that do not reside in the parts of the system, but rather in the collective. These properties cannot be found in an individual state of the system but rather in the ensemble or in the relationship of a system to the environment. Our discussion suggests that there is no strong emergence when the system is defined by constraints that act on each component. Only when constraints are defined that act on collectives and not on components does strong emergence occur. This discussion clarifies some aspects of weak and strong emergence. Although traditionally strong emergence has been considered as separate from scientific inquiry, we have shown here that there are forms of strong emergence that can be formalized through the mathematical study of multiscale variety. This recognition provides an opportunity to characterize differences between systems that are constructed from components and those that have evolved through selection. In this sense, it raises anew some issues of whether reductionism and mechanistic perspectives are sufficient to understand all complex systems. To the extent that mechanistic perspectives are characteristic of machines, this validates historical challenges to the mechanistic view of natural systems. However, to the extent that the claim is that strong emergent properties are not accessible to scientific inquiry, the formalization presented in this article suggests otherwise.

The recognition of strong emergent properties and the multiscale formalism presented here provides an opportunity for understanding how certain properties of a system can be characterized. In particular, it shows how strong emergent properties manifest in the response dynamics of a system as described by a fluctuating multiscale variety. In addition, this finding extends our ability to appreciate the subtlety of understanding different forms of complex systems.

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