The Biotic Pattern of Prime Numbers Supports the Bios Theory of Creative Evolution from Radiation to Complexity

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Mathematical analyses reveal life-like (biotic) creativity and harmonic features in the prime number series. Bios is a pattern characterized by causal novelty (measurable as reduction in recurrence by randomization). Biotic features of novelty, causation, diversification, non-random complexity, and episodic patterning are observed (after detrending) in the series of prime numbers, the differences between consecutive primes, and the distribution of primes. Plots of the sine versus cosine transforms of the primes show circular symmetry as in electromagnetic radiation, which is also demonstrable in heartbeats and in the Schrödinger equation. Recursions of sinusoidal functions generate bios. Bios has been demonstrated in quantum, cosmological, meteorological, cardiac, population, and economic series. These results support the Bios Hypothesis that natural and human processes and their evolution are causal creative processes, largely driven by cyclic engines homologous to electromagnetic radiation which are capable of generating diversity, novelty and complexity.

1 Introduction

Here I study the prime numbers with methods developed to analyze empirical creative processes. In addition to the intrinsic mathematical interest of prime numbers, they also are the exemplar of novelty, in the sense that they are not multiples of any other, and they are generated mathematically, not by random chance. Natural and human processes are creative, i.e. they generate diversity, novelty and complexity. Yet, they are described as deterministic or indeterministic. From quantum mechanics and the origin of cosmological non-uniformities to biological evolution and socioeconomic processes, "chance" is explicitly assumed to account for novelty. This fails to provide methods to foster healthy creative processes and to hinder the emergence of destructive ones. The goal of our research program is to contribute to the development of a science of creative processes and its application to human endeavors.

Table 1. Non-Periodic Processes

	Stationary Stable standard deviation. Recurrence. Stable degree of complexity. Flat power spectrum. Uniform, stable pattern.	**Creative** Diversification. Novelty. Complexification. 1/f power spectrum. Continual transformation.
Stochastic Random difference between consecutive terms.	**RANDOM**	**RANDOM WALK**
Causal ("deterministic") Patterned series of differences between consecutive terms. Partial auto-correlation.	**CHAOS**	**BIOS**

Taking biological processes as a model for creativity, we have developed a series of methods that allow one to measure diversification, novelty and life-like complexity in time series [Sabelli and Abouzeid, 2003]. These methods

identify Bios as a distinct pattern [Kauffman and Sabelli, 1998]. Biotic patterns are generated by the recursion of trigonometric functions such as A(t+1) = A(t) + g * sin(A(t)) that involve action (recursion), harmonic opposition (sine function) and conservation (the A(t) term). Table 1 summarizes the similarities and differences between biotic processes and random, chaotic and stochastic processes as measured by time series analyses.

Biotic patterns are found at all levels of organization:

(1) Quantum physics: Schrödinger wave function [Sabelli and Kovacevic, 2006] and its relativistic version, the Klein-Gordon-Fock equation [Thomas *et al*., 2006].

(2) Cosmology: temporal distribution of galaxies [Sabelli and Kovacevic, 2006] and quasars [Thomas *et al*., 2006].

(3) Planetary: air and ocean temperature, river levels, river and shore fractal forms [Sabelli, 2005].

(4) Molecular biology: sequences of bases in DNA [Sabelli, 2005].

(5) Physiology: heartbeat series and respiration [Carlson-Sabelli et al, 1994; Sabelli and Carlson-Sabelli, 2005].

(6) Biology: population size of several animal species [Sabelli and Kovacevic, accepted for publication].

(7) Economic series [Patel and Sabelli, 2003; Sugerman and Sabelli, 2003; Sabelli, 2005].

(8) Music [Levy et al, 2006].

2. Methods

Following the notion that physical action, bipolar opposition and causal creativity are basic features of fundamental natural and human processes, Bios Analysis involves three steps:

1. Measuring **Action**: Everything is a process, i.e. a sequence of actions (energy change in time). Processes thus have numerical properties, cardinal and ordinal. Data must then be collected and interpreted as time series, and their analysis must include measures of quantity (changes of intensity in time) and of order, such as directionality (temporal asymmetry) and transitivity (causality) between consecutive terms. As linear action often predominates, detrending by subtracting from each term the local average may be required to detect periodic and creative patterns. Samples of prime numbers up to two million primes are studied. Each series is compared with surrogate copies randomized by shuffling, and with random integer series sorted in ascending order, random data, brown noise, 1/f noise, and chaotic and biotic series generated with the process equation A(t+1) = A(t) + g * sin(A(t)); these series are chaotic for g between 3.9 and 4.6035…, and biotic for larger g.

2. Measuring **Opposition**: Every process includes opposite actions and sub-processes; fundamental processes such as radiation and natural macroscopic cycles embody simple harmonic opposites, i.e. bipolar, bidimensional and diverse. When only one time series is available, the opposite processes that produce it can presumably be brought to light by generating and examining the time series of the sine and the cosine of the data. **Complement plots** [Sabelli, 2000) are constructed by plotting the sine and the cosine of each term in orthogonal axes, and connecting successive terms with lines; presumably, as sine and cosine are complementary opposites, the pattern observed in complement plots of a time series reveals the opposites contained in the process that generates it. The chords form various patterns that can be readily identified.

3. Measuring **Creativity**: Natural processes are creative, i.e. generate diversity, novelty, and complex patterns. Regarding the pattern of heartbeat intervals as a model, we characterize **biotic complexity** by (1) diversification, (2) episodic complexes, (3) novelty, and (4) 1/f power spectrum [Patel and Sabelli, 1993], four fundamental properties of bios absent in chaotic attractors, (5) fractality, present also in chaos, and (6) lower entropy (Schrödinger, 1945; Prigogine, 1980]; entropy decreases in the transition from chaos to bios in mathematical recursions (Sabelli, 2005).. These time series properties are quantified with the Bios Data Analyzer (Sabelli *et al.* 2005] and the Bios Analyzer [http://www.inverudio.com].

Diversification is the increase in variance with increasingly larger samples (global diversification) or embedding (local diversification); one constructs vectors of increasing length starting with each term of the series, averages the S.D. of vectors of equal length, and plots these results as a function of the embedding, Sabelli and Abouzeid, 2003]. Diversification is observed in biotic series and in Brownian noise but not in random, periodic or chaotic series. Recurrence plots demonstrate transformations of pattern, and portray scale-free fractal structure, with distinct clusters of recurrences (**complexes**). **Novelty** in time series can be detected and measured by the quantification of recurrences of isometric vectors of N consecutive terms (N =2, 3,… 200). In biotic series and in Brownian noise, there are less recurrences than in their shuffled copies, in contrast to periodic and chaotic series that have a larger number of recurrences than randomized copies [Sabelli, 2001]. As repetition characterizes static order, I define novelty as lower than random recurrence. Novelty is the defining feature of bios that differentiates it from chaos. Statistical **entropy** measures diversity and symmetry (not disorder, as a series and its shuffled copy have the same entropy). These two components of entropy can be separately quantified by calculating entropy with a range of bins (2 to 100) and plotting it as a function of the logarithm of the number of bins; the slope measures diversity (slope) and the intercept at two bins measures symmetry (1 for symmetric data) [Sabelli et al, 2005].

3. Results

3.1 Recurrence Plots and Quantification

Recurrence plots of detrended prime series (figure 1) reveal distinct clusters of isometries (complexes) separated by recurrence-free intervals. These complexes display complex internal structure regardless of scale, as observed with fractal series. Shuffling largely obliterates complexes.
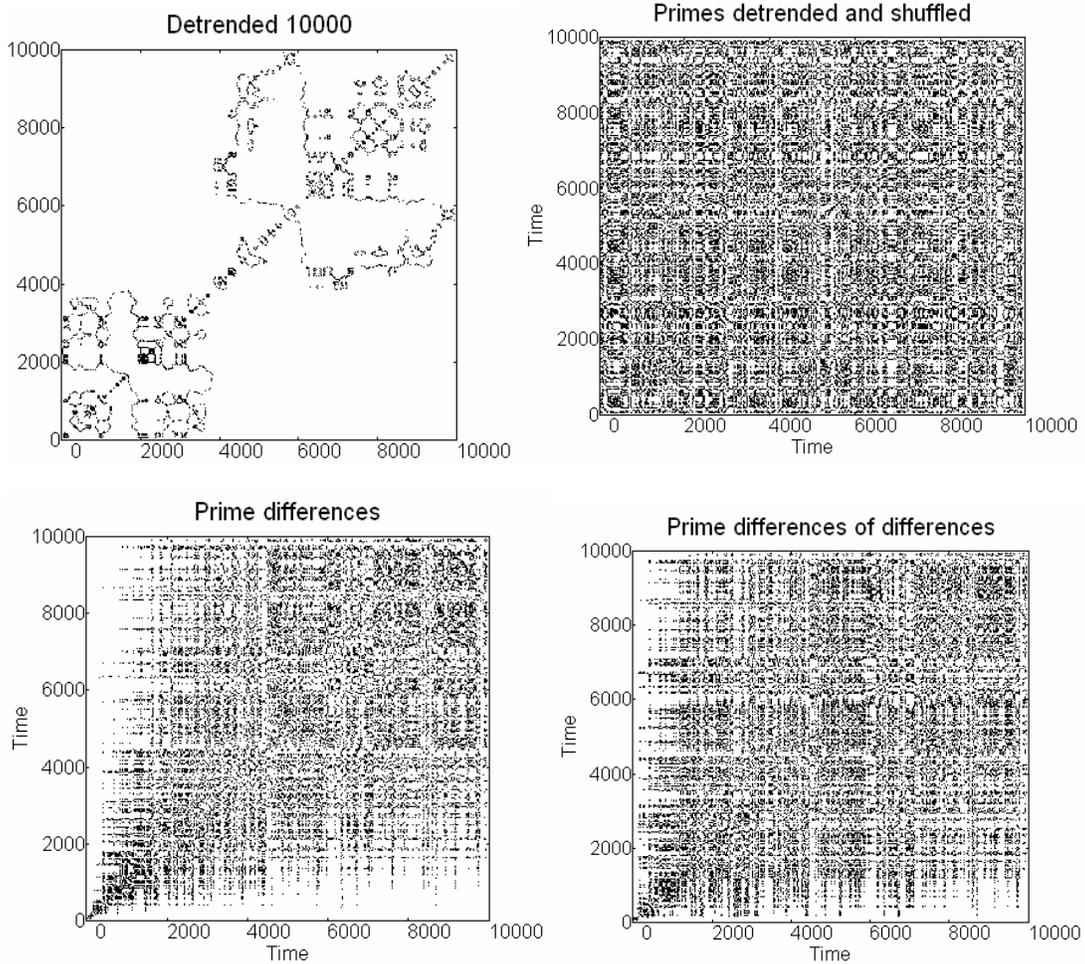

Figure 1. Recurrence plots of the detrended first 10000 primes,
50 embeddings, cutoff radius 1. 300,000 comparisons

Recurrence plots of the series of differences $\Delta P(n) = P(n+1) - P(n)$ between consecutive primes, the series of differences between consecutive primes $\Delta P(n)$ after detrending, and of the series of differences of differences, also show marked structure. Shuffling blurs and even erases completely such organization.

The demonstration of these multifarious patterns points to continually creative processes; complexes are observed in biotic series and in brown noise, while random and chaotic series show more uniform recurrence plots, similar to those of their shuffled copies [Sabelli, 2005]. Pattern in the series of differences implies that the changes that generate the process are causal actions, not random events. The creative process is causal, i.e. biotic not stochastic.

The number of isometric recurrences (figure 2) is lower in detrended prime series, the series of differences between consecutive primes, the series of differences between consecutive primes after detrending, and the series of differences of differences, than in their shuffled copies.

As action is the integral of changes of energy in time, also the series generated by the sum of differences between consecutive primes $A(t+1) = A(t) + \Delta P(n)$ and the sum of the differences of differences [after detrending the prime series $P(n)$ or the series of differences $\Delta P(n)$] have been examined, and found to show marked structure (figure 3). Novelty is clearly demonstrated in all these series.
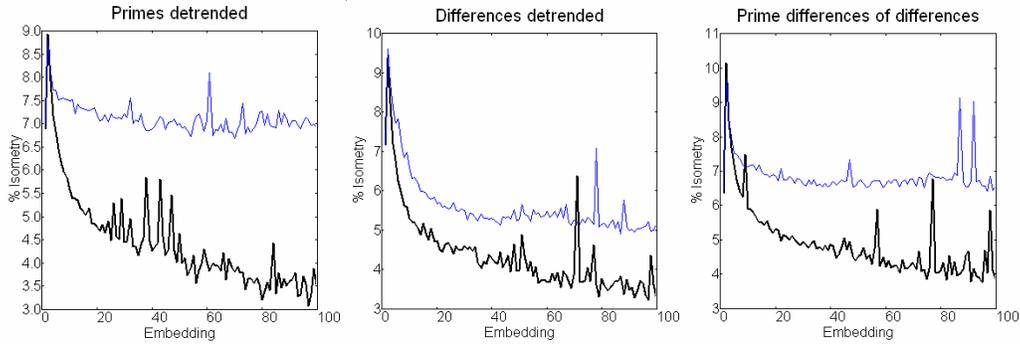
Figure 2. Embedding plots of the detrended first 3000 primes.
Thick lines show the results obtained with the original series. Thin blue lines show results after randomization.
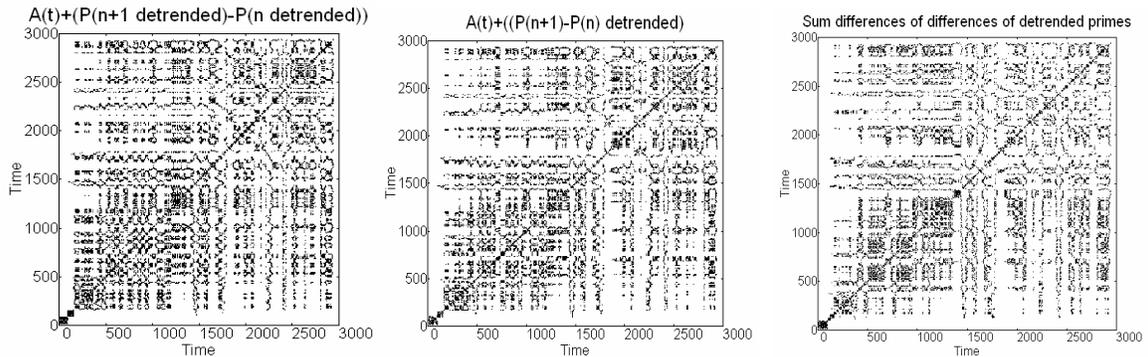

Figure 3. Recurrence plots of the series $A(t+1) = A(t) + \Delta P(n)$ (left and center) and the series $A(t+1) = A(t) + \Delta P(n+1) - \Delta P(n)$ (right) after detrending the prime series (left and right), or the series of differences (center).

3.2 Biotic statistics

Obviously variance increases in the ascending series of primes, but also detrended series display global and local diversification (figure 4), while a shuffled copy of the data has a uniform S.D. as observed with chaotic and random data.
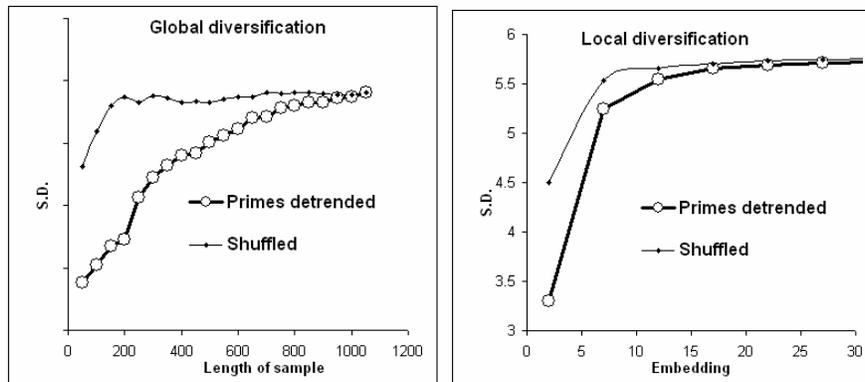

Figure 4. Increase variance with increase number of data points (left) or with embedding (right)

Prime number series have much lower entropy than random data. The entropy-bin regression lines (figure 5) show that the series of detrended primes and of the series of the difference between consecutive terms are asymmetric (intercept less than 1) as observed with creative processes (bios, Brownian, 1/f "noise") while periodic, chaotic, and random series are symmetric.

Distributing the series of primes in equal bins generates an irregular series. Recurrence plots of this series show complexes with intricate forms.
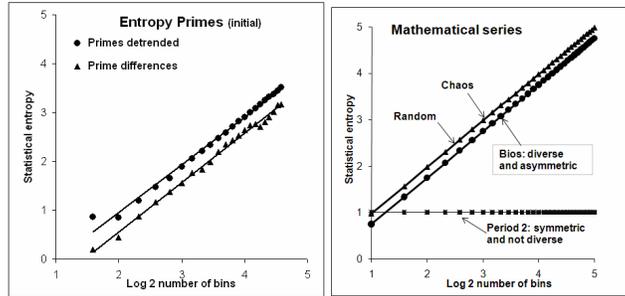
4

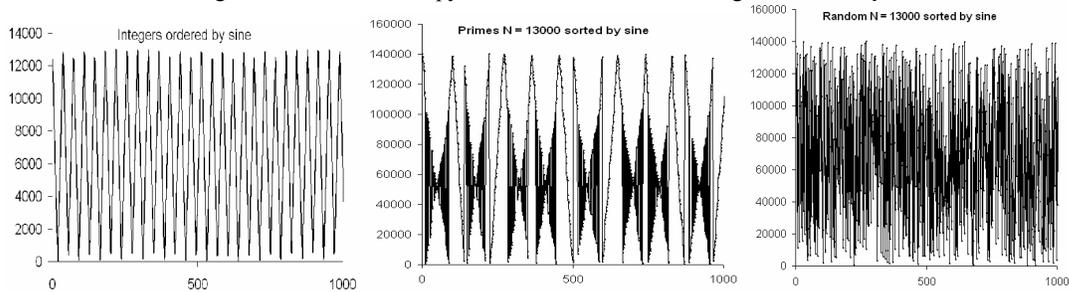Figure 5. Statistical entropy calculated with an increasing number of equal bins.

Figure 6.

3.3 Trigonometric Analysis

Plotting the series of prime numbers modulo 2 π, as well as trigonometric transformations of the primes (figure 6), show periodic features, indicating that harmonic processes may contribute to the generation of prime numbers. In particular, plotting opposite trigonometric transforms, sine versus cosine, reveals a remarkable pattern of multiple circles that many persons spontaneously describe as a Mandala (figure 7).

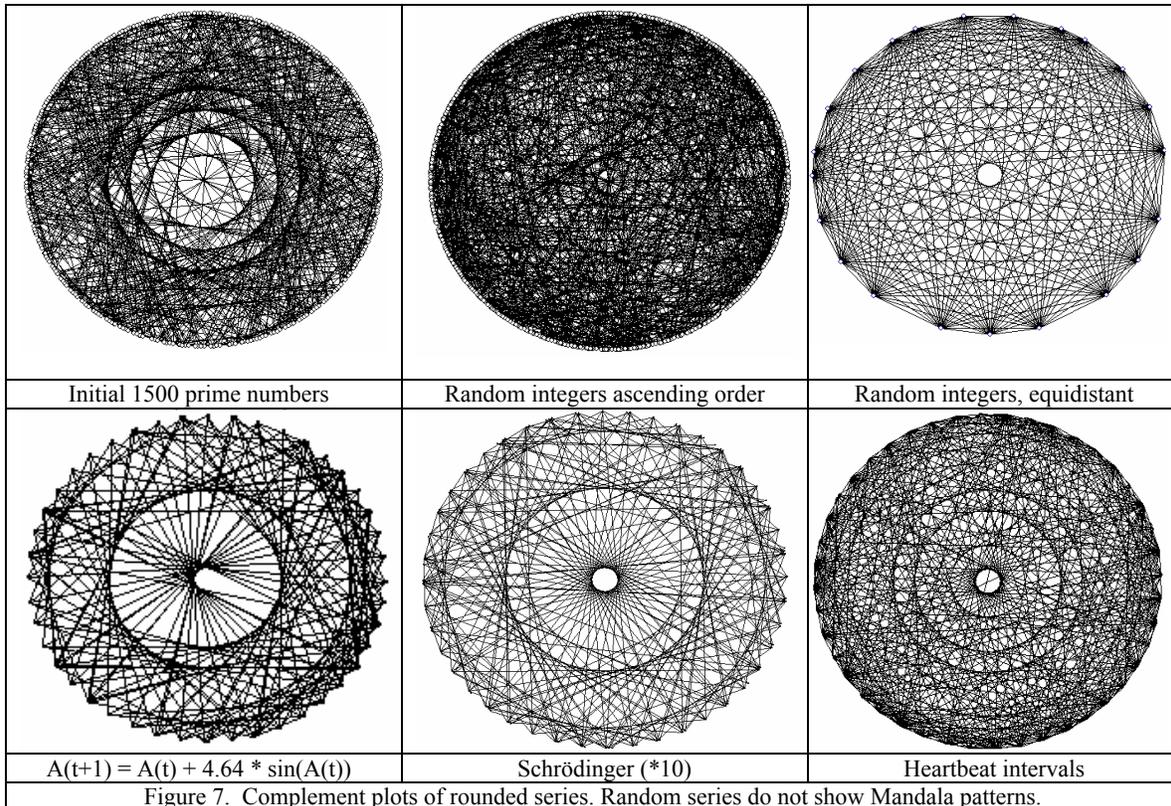| Initial 1500 prime numbers | Random integers ascending order | Random integers, equidistant |
| A(t+1) = A(t) + 4.64 * sin(A(t)) | Schrödinger (*10) | Heartbeat intervals |

Figure 7. Complement plots of rounded series. Random series do not show Mandala patterns.

4. Discussion

4.1 Biotic pattern of the prime series

The combination of novelty, complexes, diversification, entropic asymmetry and Mandala symmetry observed in the prime series is characteristic of biotic patterns generated by the recursion of trigonometric functions, including the Schrodinger equation. The prime series also demonstrates 1/f power spectrum, fractality and anti-persistence [Wolf, 1989, 1997, 1998; Shanker, 2006], features also evident in biotic series generated by the recursion of trigonometric functions [Patel and Sabelli, 2003]. These features can also be observed in stochastic noise generated by random changes but prime numbers are generated mathematically and the demonstration of pattern in the series of differences and differences of differences also implies causal rather than aleatory change. We have also found biotic behavior in series generated by the Riemann's zeta function [Kauffman and Sabelli, This Meeting].

The results obtained not only reveals biotic features in the pattern of the prime number series but they also provide evidence for a biotic process in their generation. Prime numbers appear to be generated by a cumulative harmonic processes such as those that generate bios in mathematical recursions. Mathematical experiments with recursions indicate the need for bipolar and bidimensional opposition and for the accumulation of change to generate bios; e.g. $A(t+1) = A(t) + g*sin(A(t))$ produces bios while $A(t+1) = g*sin(A(t))$ generates only chaos [Sabelli and Carlson-Sabelli, 2005].

4.2 Number, Physics and Novelty

The biotic series of primes is generated within the cumulative process of adding 1 to each successive member of the number series. A number of studies connect number theory, and in particular the prime series, with fundamental physical processes, as illustrated by the work of Hugh Montgomery, Andrew Odlyzko, and Alain Connes. Physical processes by necessity "obey" the universal laws of mathematics. Numbers are not only the product of human minds but they abstract basic properties of physical action. Action is the integral of changes of energy in time. In the prime series, the homolog of temporal order is sequence, and the homolog of action is the change in the difference between consecutive primes and the differences of differences. As illustrated by recursions that accumulate these changes (figure 3), the generation of primes shows a biotic pattern (once the linear component of change in the prime series is removed by detrending). As discussed in the introduction, also physical, biological and human processes display bios.

Action is cardinal (intensity of energy), ordinal (time) and arithmetic (sum or integral). Change is always a multiple of action units (quanta). Numbers have three aspects: ordinal, cardinal, and formal (as highlighted by Pythagoras, Pierce, and Gödel). Ordinal, cardinal, and formal are not three series of numbers, but three aspects of each number. Order and quantity, ordinals and cardinals, grow together in the linear sequence of numbers. By necessity accumulation (cardinality) requires increase in time (order). The accumulation of change generates novelty and complexity; the sum of random change generates complex Brownian noise, and the sum of chaotic events generates Bios. The mere sequence of numbers generates patterns, starting with the alternation of odd and even numbers (period 2); period 2 is a universal dialectic of nature. It also generates the prime numbers, each representing a new form. Primes are examples of novelty, complexity and nonlinear changes in quality associated with changes in linear quantity.

The law of quantity and quality posits a general relation between linear and nonlinear processes: changes in linear quantity generate nonlinear changes in quality, which may be gradual and linear or abrupt and nonlinear (e.g. biological thresholds and physical critical points). Linear and non-linear changes are associated. as complementary aspects of processes, not two different classes of processes. Pythagoras was perhaps the first scientist to show the relation between quantity and quality, in his empirical study of the relation between the length of the chords and harmony between their musical sound. Galileo famously stated that the book of nature is written in the language of mathematics, and highlighted the importance of scaling in physical processes. Scaling is fundamental in biology [Brown and West, 2000]. Hegel advanced the notion that quantitative change becomes qualitative change (e.g. water into ice) that Engels generalized as a law of nature. Expanding this concept, primes show that **linear increases in quantity create novelty, quality and complexity.** The law of quantity and quality provides a method to analyze and promote creative processes.

The prime number theorem relates the three aspects of number: the frequency of novelty (density of primes) decreases with the increase in their ordinal and cardinal value. On the other hand, the prime number series is fractal. This coexistence between numerical dependent and independent quality exemplifies a complementarity of opposites such as observed in harmonic processes.

The sequence of integers may be regarded as period 1. Each prime number is the origin of the infinite series of its multiple harmonics. In turn, the process that generates primes appears to involve harmonic functions [Shlesinger, 1986; Ares and Castro, 2006]. The zeros of the Riemann zeta-function can be regarded as harmonic frequencies in the distribution of primes. Here we observe harmonic features in the prime series itself. In particular, plotting opposite trigonometric transforms, sine versus cosine, reveals a pattern of multiple circles, a two-dimensional view of the helical motion of the trajectories of the prime generating process in the third, here collapsed, dimension of time. Prime

generation involves rotation. Such pattern of multiple circles is found in few but significant processes: heartbeat intervals [Sabelli, 2000] and in integer transforms of series generated by the Schrödinger and the Klein-Gordon-Fock equations [unpublished]. The pattern is also displayed by integer transforms of sine waves and of biotic series generated by recursions involving sine waves but not in other similar series.

Many persons spontaneously describe this pattern as a Mandala, an ancient religious symbol found in almost all civilizations. Young children spontaneously draw Mandala forms. They also appears in dreams and doodles of adults [Jung, 1967]. Mandala forms occur also in E8 [http://aimath.org/E8/mcmullen.html]. The appearance of the same form in physical and biological processes, psychosocial symbols and mathematical series points to deep homologies in the universe.

4.4 Bios as a primal process

From harmonic motion and electromagnetic radiation to proteins and DNA, helical patterns are fundamental. The sinusoidal pattern of electromagnetic radiation embodies a mutual transformation of opposites that may impart this dialectic to more complex physical and mental processes. The widespread occurrence of Bios is to be expected as there are multiple recursive processes in nature that involve cycling starting with electromagnetic radiation that construct atoms, molecules, and organisms, and transmits information across cosmological distances as well as in atoms, computers and brains. There are likewise cyclical processes at higher levels of complexity, including biological, economic, and social where bipolarity would imply synergic and conflictual interactions, as contrasted to one-sided notions of competition and struggle as the motor of biological evolution and economic development. According to Bios Theory, these synergic and conflictual interactions are cyclic engines that drive motion and qualitative change.

As radiation embodies harmonic opposites, recursions of harmonic functions generate biotic novelty, diversity and complexity, and bios is ubiquitous in nature, I conjecture that homologous processes of harmonic feedback causally generate complexity at all levels of organization. The analysis of primes extends previous observations on physical, biological and socioeconomic processes to the most basic mathematical structure of reality.

Physics embodies number, and human minds create number, pointing to a profound homology between levels of organization. Such homology is illustrated, for instance, by the Mandala created by sinusoidal waves as radiation, by the prime number series, and by personal and religious mental processes. As number series abstract fundamental properties of action, and the generation of primes epitomizes causal novelty, the results support the notion that causality is creative, not deterministic. Creation is causal, not aleatory. I conjecture that causality and creativity in nature are mathematically determined [Sabelli, 2005].

The biotic pattern of prime number series points to the role of cumulative changes in the generation of novelty, and the patterns observed in their trigonometric transformation to harmonic processes. These results are consistent with the Bios Hypotheses:

1. Cumulative increases in quantity and order create novelty, quality and complexity. This process is universal, because physical action is cumulative (action is the integral of energy changes) and asymmetric (action involves temporal order).

2. Fundamental natural and mental processes are harmonic, i.e. involve bipolar, diverse, bidimensional, and mutually transforming (not mutually excluding) opposites. In other words, they contain cyclic engines that drive motion and qualitative change. Such harmonic opposite processes are not portrayed appropriately by traditional, dialectic or Boolean logic, but can be represented by trigonometric functions. This bios model of opposition may then be useful to model logical negation in quantum computation [Sabelli and Thomas, in press].

3. Biotic processes combining cyclic engines and accumulation are ubiquitous in natural and human processes, and may play a significant role in physical and biological evolution [Sabelli, 2007].

The analysis of primes thus grounds Bios in Number Theory, and in turn points to dialectic features in the logic of mathematics. These notions are pursued in a companion paper [Kauffman and Sabelli, This Meeting].

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