

Scale Free Networks; A Literature Review

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The Scale-Free Networks model is considered a significant discovery because it has been successfully applied to many complex real-world networks and proved valid. The successful application of the recently discovered model deemed the other model, the random network model, questionable. The presence of scale-free emerging properties in many “real-world” networks provides initial evidence that these self-organizing phenomena do not only depend on the characteristics of individual systems, but are general laws of evolving networks. This research studies the “BA scale-free network model” in detail and discusses a variety of its applications. It provides a literature review for many of the discovered scale free networks in nature to the best of our knowledge. It also suggests the exploration of data, information, or knowledge flow relationships among members of work groups in the virtual Information Technology project networks with dispersed team members working in different countries/locations in the world because to the best of our knowledge, this network hasn't been studied from a scale-free network point of view despite its importance.

1 Introduction

In the past few years, we were able to recognize networks everywhere ranging from networks in nature to networks of people. The spectrum also contains planned networks designed by human beings as well as unplanned networks that naturally evolve and self-organize into complex forms that are discovered by researchers and scientists after they are already part of our everyday life. This makes planned networks easy to map because we know the number and location of nodes and the connectivity pattern in advance, and makes unplanned networks difficult to map because they need to be discovered or felt after they are established.

Denning (2004) gave some examples of networks that we can recognize in our everyday life and everywhere such as: air traffic networks, banking networks, chemical bonds networks, data communications networks, ecosystems networks, finite element grids, fractals, interstate highways, journal citations, material structures, nervous systems, oil pipelines, organizational networks, power grids, social structures, transportation networks, voice communication networks, water supply networks, and Web URLs.

Wang and Chang (2003) defined the nodes that represent the building blocks of complex networks as follows: The Internet is a network of routers or domains. The World Wide Web (WWW) is a network of websites. The brain is a network of neurons. An organization is a network of people. The global economy is a network of national economies, which are themselves networks of markets. Markets are networks of interacting producers and consumers. Food webs and metabolic pathways can be represented by networks. The relationships among words in a language can be represented by a network. Topics in a conversation can be represented by a network. Even strategies for solving mathematical problems can be represented by a network. Diseases like AIDS are transmitted through social networks. Energy is transmitted through transportation networks such as power grids.

Many of the most recent studies in networks analysis are interested in understanding how the “complex network structure” facilitates and/or constrains the network dynamical behaviors, which have been largely neglected in the studies of traditional disciplines. Those efforts to understand complex networks have been aided much by the computerization of data acquisition and the availability of high computing power because the availability of huge databases on various real networks of complex topology became possible. The super computational power and detailed topological information about very large-scale real-world networks led to the recent success in uncovering the generic properties of different kinds of complex

networks. The efforts became more realistic and made more sense with the aid of the most significant recent discoveries, which are the small-world effect and the scale-free feature (see literature review) of most complex networks.

In 1998, physicist Albert-Laszlo Barabasi and his colleagues at the University of Notre Dame used a Web Crawler to map the Connectedness of the Web. Surprisingly, they discovered that the structure of the web doesn't follow the random connectivity model widely accepted then. They called the connectivity map that resulted from their experiments a "scale-free" map. The notion of scale-free networks has turned the study of a number of networks upside down. A simple definition of scale-free networks is the network whose nodes aren't randomly or evenly connected, but includes many "very-connected" nodes known as the hubs of connectivity responsible for shaping the way the network operates. A scale-free network doesn't have a fixed size but can grow with time. The ratio of the "very-connected" nodes to the number of nodes in the rest of the network remains constant as the network's size changes.

On the other hand, the widely accepted random connectivity models used before Barabasi made his discovery postulated that there would be "no" well-connected nodes or "very few" statistically insignificant well-connected nodes. In such random networks most nodes would have a number of connections hovering around a small mean value. Also, as the number of nodes in a random network increases, the relative number of very-connected nodes decreases.

The scale-free networks belongs to the family of networks known as "small-world" networks that are based on the notion that there are only 6 degrees of separation between any two people in the world, and thus, it doesn't take many hops to get from one node to another in such networks. Therefore, in a scale-free network, there is a high probability that many transactions would take place through one of the well-connected nodes known as the hubs of the network. Taking the World Wide Web as an example, more daily transactions take place through Web hubs like Yahoo Web portal and Google Web portal than those which take place through other less-connected Web portals.

The scale-free networks break down in a way different from that of random networks. When nodes are removed from a random network, the connectedness of the random network decays steadily and slowly with time until the network reaches a point where it breaks into smaller separate domains that are unable to communicate. On the contrary, scale-free networks resist random failures. This is because it is statistically unlikely that the very-connected nodes would fail under random conditions and the very-connected nodes are the ones responsible for the scale-free networks failure. Thus, connectivity in the scale-free networks is maintained under random conditions. A scale-free network completely fails only when the hubs are wiped out and this can only happen after a lot of random failures. Resisting random failures is the good news. However, the catastrophic failure of scale-free networks under targeted attacks is the bad news. This happens when worse is directed at hubs. If all the very-connected nodes are taken out of the network, the network stops functioning immediately. Therefore, it can be safely concluded that scale-free networks are very dangerous under targeted attacks. Thus, in a network that follows Barabasi's model, stopping failure from spreading requires that one focuses on protecting the hubs and not the many thousands nodes forming the network.

2 Literature Review

We have reviewed the literature to put together and summarize some of the work done by others with respect to scale-free network topology in different kinds of networks ranging from communication networks like the World-Wide Web to biological networks like the human brain. The following represents the findings of this literature review:

Albert et al. (1999) studied the World-Wide Web (WWW) as a complex network that grows in an uncontrolled manner as any individual or institution can create a website with any number of documents and links. The unregulated growth leads to a huge and complex web, which becomes a large directed graph whose vertices are documents and edges are links (URLs) that point from one document to another. The results show a power-law with a significant probability of finding documents with a large number of links because the network connectivity is dominated by highly connected web pages. The results also show that the web forms a small-world network, as the average diameter of the web is only 19 links. Any two randomly chosen documents on the web are on average 19 clicks away from each other.

Eguiluz et al. (2004) used functional magnetic resonance imaging to define networks among correlated human brain sites. The analysis shows that the distribution of functional connections and probability of finding a link vs. distance are both scale-free. There is a relatively large proportion of nodes with a single link, and a smaller proportion of nodes with numerous connections. This result implies that there is a small but finite number of brain sites that have broad access to most other brain regions. Those well-connected nodes are comparatively much more numerous than in a randomly connected network. The properties discovered by Eguiluz et al. can, in their opinion, be very useful to investigate the dynamics of brain states, especially in dysfunctional cases.

Newman empirically studied the time evolution of scientific collaboration networks in physics and biology. In these networks, two scientists are considered connected if they have coauthored one or more papers together. He showed that the probability of scientists' collaboration increases with the number of other collaborators they have in common, and the probability of particular scientist acquiring new collaborators increases with the number of his/her past collaborators. His results support clustering mechanism and power-law degree distribution in networks.

Barabasi et al. (2002) studied the co-authorship network of scientists and found that it represents a prototype of complex evolving networks. They captured the dynamic and the structural mechanisms that govern the evolution and topology of this complex system by mapping the electronic database containing all relevant journals in mathematics and neuro-science for an 8-year period (1991-98). This represents the largest publicly available social network. The network constantly expands by the addition of new authors to the database and the addition of new internal links representing papers co-authored by authors that were part of the database, as well as external links, links representing papers for authors that were not in the database. The fraction of the links created as internal links is much higher than those created by the incoming nodes because an author qualifies for a new incoming link only on his first paper, while most scientists contribute for a considerable time to the same field by publishing numerous subsequent papers that appear in the network as internal links. Thus, the network's topology is much more driven by the internal links than by external ones. The results indicate that the network is scale-free, and that the network evolution is governed by preferential attachment.

Liljeros et al. (2001) studied the web of human sexual networks. They have chosen this network in particular because it has "unambiguous" type of connection. They analyzed the data gathered in a 1996 Swedish survey of sexual behavior for people aged 18-74 years. Their results reveal that the cumulative distribution of the number of different sexual partners in one year decays as a scale-free power law that has a similar exponent for males and females. Because the network is dynamic, the number 'k' of sex partners over a relatively short time period (the twelve month before the survey) was analyzed. (i.e. connections in the network appear and disappear as sexual relations are initiated and terminated, and thus, there exist some short-lived links). Males report a larger number of sexual partners than females, but both have the same scaling properties. They attributed this scale-free behavior to three main reasons. The first is the increased skill in acquiring new partners as the number of previous partners grow. The second is the varying degrees of attractiveness. The third is the motivation to have many new partners to sustain self-image. The results support the preferential-attachment mechanism of scale-free networks, and the "rich-get-richer" concept.

Dezso and Barabasi (2002) studied the unstoppable viruses' diffusion and spreading on a scale free network. This includes both biological and computer based viruses. Most methods designed to eradicate viruses aim at reducing the spreading rate of the virus. Even when the virus has a zero epidemic threshold, while a reduced spreading rate will reduce the virus prevalence, there is a little guarantee that it will eradicate it. This is because in scale free networks, the hubs are in contact with a large number of nodes, and are therefore easily infected. Once infected, they pass the virus to a significant fraction of nodes in the system. Thus even weakly infectious viruses can spread and prevail on a scale-free network. This negates what diffusion studies used to believe prior to the scale free network theory that viruses whose spreading rate exceeds a critical threshold will persist, while those under the threshold will die out shortly. Dezso and Barabasi argued that hub-biased curing policies (curing with higher probability the hubs than the less connected nodes) can restore the epidemic threshold, which can stop the virus spreading.

Jeong et al. (2001) studied protein-protein interactions and expanded the protein's primary roles as catalysts, signaling molecules or building blocks in cells and micro organisms into the new role as an element in a network of protein-protein interactions in which it has a contextual or cellular function. They gave quantitative support for this idea. They showed that the robustness against mutation in yeast is also

derived from the organization of interactions and the topological positions of individual proteins. They also showed that the most highly connected proteins in the cell are the most important for its survival (Jeong et al. P.41).

Lusseau (2003) studied the topology of the animal social network represented in the community of bottlenose dolphins. Using a dataset consisting of 64 individuals, he found that the connectivity of individuals follows a complex distribution that has a scale-free power law distribution for large k . The network is characterized by the presence of “centers” of associations which means that not all individuals have an equal role in the society. The hubs represent adult females. Those hubs are responsible for maintaining a short information path between individuals of the population. Another important finding is that the ability for two individuals to be in contact is unaffected by the random removal of individuals. Unlike other scale-free networks, the removal of individuals with many links to others (targeted attack) only affects the information path between two individuals, but it does not fragment the cohesion of the social network. The network survived a targeted attack (with the removal of 20% of individuals) and formed a large cluster encompassing most individuals [typically, scale-free networks become fragmented into small clusters under targeted attacks as shown by Albert et al. (2000)]. Even with the removal of 30% of individuals (randomly or selectively), the network formed a large cluster that encompasses most of the individuals present and some single individuals without any associates. The survival of targeted attacks means that hubs (individuals with many companions) do not maintain the cohesion of the network. The self-organizing phenomenon helps in overcoming catastrophic death events that would result in the loss of more than a third of the population by keeping the network remain united.

Onody and Castro (2004) studied the complex network of Brazilian soccer players with nodes representing soccer players and clubs (13,411 soccer players and 127 clubs). The results show that the probability that a Brazilian soccer player has worked at N clubs or played M games shows an exponential decay, while the probability that a player has scored G goals follows a power law distribution. He also studied another network composed of players who have worked at the same club at the same time. The results show that the degree distribution of this network decays exponentially.

Riccaboni and Pammolli (2001) studied the firm growth on networks because the body of knowledge in networks field have shown that network structure and positions in networks influence firm performance and growth, which affects market structure. They were particularly interested in networks of contractual relationships among firms specialized in research and exploration (originators) and firms focused on development, production, and commercialization (developers). They studied processes of internal growth and processes of external growth through collaborative agreements. They showed that “these networks’ growth is shaped by entry of new firms and by proportional growth of the connectivity of individual firms, with remarkable departures from the regime of universal random growth” (Riccaboni and Pammolli, P.3).

Braha and Bar-Yam (2004) studied the statistical properties of networks of people engaged in distributed problem-solving and product development activities. They were able to show that problem-solving networks have properties (sparseness, small world, scaling regimes) similar to those of information, biological and technological networks. They found that complex PD (product development) networks display similar statistical patterns to other real-world networks of different origins. They concluded that “cutting out” some of the links (eliminating some arcs) between the tasks can accelerate the product development activities, but this might worsen the performance of the end system. Thus, a tradeoff should exist between the elimination of task dependences and the desire to improve the system’s performance through the incorporation of additional task dependencies. Finally, the highly connected nodes of the scale-free design network tend to be the most reusable modules in future product architecture. This allows firms to reduce the complexity and scope of the product development project by embarking upon the knowledge embedded in reused modules. This results in significantly reduced product development time.

Garlaschelli et al. (2004) described a network of large market investments, where both stocks and shareholders are represented as vertices connected by weighted links corresponding to shareholdings. In their framework, the in-degree (k_{in}) correspond to the number of assets held (portfolio diversification) and the sum of incoming link weights (v) of an investor corresponds to the invested wealth (portfolio volume). The empirical analysis of three different real markets reveals that the distribution of both (k_{in}) and v display power law tails.

Bagler (2004) studied the airport network of India (ANI), which represents India’s domestic civil aviation infrastructure, as a complex network. His research concluded that India’s network of domestic

airports, connected by air links, is a small-world network characterized by a truncated power-law degree distribution and has a signature of hierarchy. The traffic in ANI is found to be accumulated on interconnected groups of airports and concentrated between large airports. Large airports (hubs) provide air connectivity to far-off airports, which themselves do not tend to be connected. With increasing degree, airports have a progressive tendency to form interconnected groups with high-traffic links. This is known as “rich-club” phenomenon, in which high degree nodes tend to form cliques with nodes with equal or higher degree.

Paczuski and Hughes (2003) studied the sun as a heavenly example of self-organized criticality. Sudden bursts of intense radiation emanate from rapid rearrangements of the magnetic field network in the corona. Avalanches are triggered by loops of flux that reconnect or snap into lower energy configurations when they are overly stressed. This system represents a network in which the footpoints are the nodes and the loops are the links. The results show that the nodes and loops embody a scale-free network after attaching in the photosphere. The strength of the link between two nodes, which is the number of loops connecting two nodes, is distributed as a power law. In addition, the number of unique nodes linked to any specific node also exhibits a scale-free behavior following a power law distribution.

Boss et al. analyzed the network structure of the Austrian interbank market based on a unique dataset of the “Oesterreichische Nationalbank” (OeNB). The results came in contrast to interbank networks analyzed previously in the economic and econo-physics literature. Contract size distribution follows a power law over more than 3 decades, which can be understood as being driven by the underlying size and wealth distributions of the banks because they show similar power exponents. The degree distribution of the interbank network showed two different power law exponents which are one-to-one related to two sub-network structures differing in the degree of hierarchical organization. The existence of a power law results in a stable network with respect to random bank defaults or intentional attacks. In addition, the interbank network shows “small-world” properties represented in a low clustering coefficient that indicates a very low “degree of separation” between any two nodes in the system. In real life, this can be interpreted as follows: banks would have links with their head institution, while these few head institutions hold links among each other.

Anghel et al. (2004) argues that in a competition environment with very limited resources, people are able to make the most gains if they avoid the crowds and place themselves into the distinguished class of the elites (the few). They believe that even though this class forms a minority group when compared to the whole society, it can greatly influence the dynamics of the entire society because the elites holds the best strategies in any given situation, and thus, they become “key target nodes” for others to communicate with and follow. For them, an agent is a leader if at least one agent is following and acting on his advice. A number for a leader is measured by the number of followers he has. Agents who are not leaders are simply followers. Leaders can also follow other leaders. Following their steps, a leadership structure is created. Both the leadership structure and leaders are very dynamic because the success of a strategy depends on the context of the strategies used by the other agents. They used Reinforcement Learning as a mechanism for deciding whom or what to follow. They defined reinforcement learning as “a mechanism for statistical inference created through repeated interactions with the environment” (Anghel et al. P.1). They studied their scenario using a multi-agent model of competition known as the minority game. They modified the model to include inter-agent communications or influences across a social network. They based their model on the assumption that the precise topology of the social networks is not known, but it is known that social networks have a small world character. The results show that the pure followers constitute over 90% of the population, and that all layers of the leadership hierarchy are equally influential.

3 Research Contribution

Barabasi (2002) appreciated the efforts to map those complex real-world networks because those maps are helping to shed new light on our “weblike” universe. Detailed maps of the Internet have unmasked the Internet’s vulnerability to hackers. Maps of companies connected by trade or ownership have traced the trail of power and money in Silicon Valley. Maps of interactions between species in ecosystems showed humanity’s destructive impact on the environment. Maps of genes interaction in cell uncovered the ways of how cancer works.

However, we consider the efforts incomplete due to the presence of a considerable number of real-life networks that haven't been studied yet. This is not because scientists are not interested in studying networks, but because those new concepts and topologies had only been discovered in the late 1990-s. Although a fairly large number of complex networks have been proven to be scale-free, generalizing the scale-free networks model proposed by Albert-Laszlo Barabasi and his colleagues in 1999 is still a dilemma.

The human body is a clear and representative example for a complex network. It is a "System of Systems", or in a more accurate expression a "network of systems". The important feature about this network is that not all systems are equally important to the whole. For example, a blind person who lost his two eyes in an accident can live long enough to emphasize the unimportance of this specific system to the survival of the overall system, the human body. Although the eyes as a system are very important and they perform a very important function that contributes to the overall system's efficiency, the removal of such system from the body network affects the network but doesn't fragment the network. The Heart as a system, on the other hand, is more important to the network because it synchronizes the flow of blood to all the other systems in the network. Thus, if the heart is removed from the body, the body collapses. This is because blood no longer reaches the other systems and thus the network becomes a network of fragmented systems not talking to each other. Therefore, the "Heart" is considered one of the major players that hold the body network together. Another example of a major player in the body network is the brain. The brain controls many of the actions taken by other body organs because it is the source of signals that start those actions. For example, when a person decides to climb the stairs of a building the operation starts from the brain. The decision is made in the person's mind and then his brain sends some signals in the form of electrical energy, carried through synapses to another system which is the "legs". The legs respond by moving alternately and climbing the stairs. The order for every single action taken by any system in the body comes from the brain. Thus, the brain is a central system and a key one that cannot be removed from the network, otherwise the network collapses. Therefore, a conclusion can be made that all systems play a role in the overall network function. This means when all systems are functioning properly, the network will be optimized. However, when some systems are not functioning properly, the global optimality of the overall network can never be achieved. Also, another important conclusion is that not all systems of the human body network are of equal importance to the network. A first category is that of systems that can be easily removed from the network like the addendum with absolutely no effect on the overall network. The second category is that of the systems that affect the performance of the network or the performance of some neighboring systems in the network if removed. The third and most important category of systems is that of the ones that when removed from the network, the network collapses and is transformed into a group of fragmented nodes that are not talking to each other. This third category is a category of the critical systems that need special care and need to be monitored more closely than the others.

Our research suggests considering the people's network in a virtual Information Technology project team as comparable to the human body. The project organization represents the network of systems. Some of those systems are not important to the network. They are the weak performers that when removed from the network, the network is not affected. We categorize those as the "followers" in the network. Some other systems are very important to the network so that when they are removed, the network's performance is not globally optimized. Those are the people with a lot of experience and plenty of knowledge but of little action. If they are proactive, they can be moved to the third category of systems. This third category is the category of people who are more experienced than others, more knowledgeable than others and better performers than others because they are proactive in using and sharing their knowledge and expertise with other people in the project network. This third category is the category of "leaders". However, for simplicity in mapping the network, we suggest neglecting the category in between the "followers" and the "leaders" and considering only a network that is formed of either leaders or followers in a virtual Information Technology project team. The leaders all together represent the heart of the network as they synchronize the flow of energy to the followers leading to better performing followers. They blow energy represented in information, knowledge or data in followers. This energy provides power to followers by inspiring them to do more work. The knowledge can be a document without which a follower would not perform. It can also be tacit knowledge represented in a leader's prior experience with a similar project, operation or transaction. The leaders are the hubs of the network while the followers represent nodes that are not of the same importance as the hubs. Removing one leader from the network may not cause the

network to collapse. However, removing all leaders will definitely cause the network to stop functioning. Removing all leaders except one however may or may not cause a network failure because followers may reorganize themselves around that leader and the network is firmly held together again.

In particular, the research aims to explore if the distribution of data, information, or knowledge flow relationships among members of work groups follows a Gaussian distribution resulting from a purely random network having a mean number of relationships and a homogeneous distribution of the flow pattern around that mean value, or if it follows a scale-free topology that governs the network evolution, with the presence of a few hubs (more important nodes) responsible for connecting people together and responsible for most of the energy flow in the network, and many other tiny nodes (less important nodes) representing people who are only connected to very few coworkers and who contribute very little to the data, information or knowledge dissemination, and thus to the influence in the project's energy flow network. We also suggest exploring the failure mode of such network under inevitable random failure (the random removal of nodes from the network) and under intentional attack (the removal of specific targeted most-connected nodes) and to discover the network fragmentation pattern. This will give another clue to conclude if the network follows the ER random model or the BA scale-free model because as Barabasi concluded, a scale-free network is resilient to random failures but is fragile and fragments to several unconnected sub-networks in some cases of targeted attacks.

Our research will hopefully have an important contribution to both theory and practice. The answer to our research question will contribute to the networks body of knowledge in general. It will add to the literature some important information about the topology and the evolution process of one of the important complex networks that evolve throughout time as a non-man made network. This will hopefully add to the efforts already exerted by many researchers and scientists to understand networks, map networks, connect networks, and keep networks' connectivity healthy, whether they are man-made or naturally evolved networks. This will be a good step forward towards building a universal network theory that can classify networks easily and correctly given their characteristics.

Practically, the Information Technology virtual project team representing the leader-follower influence network will help any organization in general and large-sized and Multinational ones in particular, to run its day-to-day business. The importance of the study increases when the projects of this organization are developed offshore or are carried out in different countries of the world cooperatively. This network helps in organizing efforts within a work group as well as in the intra-project operations management. If the network fails or malfunctions, the members of the virtual project team may be working as separate entities not linked together which may lead to an inefficient project. Thus, our study will give Information Technology developers important insights that will help them keep their virtual project team dynamic network not only up and running, but also running in a healthy way. It will help organizations understand if their project network has hubs that need to be appreciated, or if all nodes representing people are equally important to the project. This can help in forecasting the self-organization behavior of those people in case of losing some nodes, and thus, predicting the timing and the extent of failure.

If the answer of our research question is that the network is a completely random one, an organization should try to know the threshold beyond which no more people can be taken off the network (where all people are equally important from influence point of view) without project failure. This will help them know the number of employees they shouldn't go beyond in a project downsizing effort for example in order to keep the virtual work group network cooperative.

On the other hand, if the answer is that work group leader-follower influence network is a scale-free network, this would have far more important implications. It would mean that the network has some hubs, and that those hubs hold the network tight. The organization shouldn't plan against the network's random failure because the network can always resist it. If some people suddenly leave the project and if some of those people are hubs, the other hubs in the network will keep it inter-connected and tight and prevent work-group network failure. The bad news then would be the vulnerability to targeted attack. Practically, this means that if many of the hubs leave the network at one time, the network might collapse forming a group of isolated non-communicating clusters of followers. A project may be targeted by some competing organizations that can attack its hubs by trying to hire many of them at once for example. When a certain number of leaders (hubs) leave a project, no "proper" leadership can take place, which will lead to a project's failure and will be reflected on the organization's operations and profitability. In short, the

organization could be paralyzed under intentional attacks. Thus, planning against those intentional attacks would be both possible and essential if this influence network is a scale-free one.

Thus, an answer to the question “Is the coworkers work groups leaders-followers influence network in a virtual Information Technology project team a scale-free network with a connectivity (nodes degree distribution) that follows a power-law degree distribution, and that can be fully represented and described by the “BA Scale-Free” model, or is it a random network with a connectivity that follows (or can be approximated to) a Poisson degree distribution, and that can be fully represented and described by the “Erdos-Renyi Random” model?” is still not presented in the networks literature, an answer that needs to be formulated via our future research efforts to understand the characteristics of this knowledge flow network and to recommend the best way to maintain it operating in a healthy manner.

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