Intractable Conflict as a Dynamical System

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We show how mathematical methods from complex systems can be used both as metaphors and mathematical equations to understand the nature of conflicts between individuals, groups, and nations and present novel methods for their resolution. The metaphorical approaches have been successful used as training materials for practical applications in conflict management. The mathematical model allows us to test the effects of specific social mechanisms in a conflict. In the model, the state of each group depends on its own state in isolation, its previous state in time, its inertia to change, and the influence from the other group. Each group can influence the other group either through positive or negative feedback. We show how the positive or negative feedback between the groups leads to different dynamics and steady states.

1 Background: The Dynamics of Conflict Consortium

Our Dynamics of Conflict Consortium is an interdisciplinary, international group of practitioners and scholars in mediation, conflict management, social psychology, complex systems, and mathematical modeling who are working closely and collaboratively together to understand the nature of conflicts and their resolution. We are working at three different levels:

- Field studies (questionnaires, interviews, social network analysis) in Africa
and the Middle East.

• Social psychology laboratory experiments with participants in New York, Boca Raton Florida, and Warsaw Poland.

• Mathematical models (differential equations, cellular automata, networks).

The novel approach used in all these studies is based on dynamical systems theory. We are showing how these concepts from mathematics can be used to develop and apply new metaphors and specific mathematical models to analyze and understand conflicts and serve as the basis for training materials in conflict management and resolution.

We present here two different tools, representing two equally valid and useful ways in which the insights from dynamical systems theory can be used to help understand conflicts. Each tool reveals, and perhaps also hides, different aspects of conflicts. The first tool uses the concepts from dynamical systems as a framework to define a new paradigm in which to organize concepts about conflicts and suggest ways of resolving them. The second tool uses a specific mathematical model whose properties resemble aspects found in conflicts and suggests new ways in which the dynamics of a conflict in time depends on the feedback between the actors in the conflict.

2 The Attractor Paradigm: Using Complex Systems Metaphors to Understand Conflicts

An attractor is a subset of potential states or patterns of change to which a system converges over time. An attractor "attracts" the system's behavior, so that even very different starting states tend to evolve toward the subset of states defining the attractor. Once at its attractor, the system is resistant to external influences that would otherwise move the system to a different state or pattern of changes. An external influence might promote a temporary change in the state of a system, but over time the system will return to its attractor. This dynamic property helps understand why malignant social relations tend to be persistent over time and resistant to intervention, despite the absence of goal attainment by either party, let alone the optimization of hedonic benefits (e.g., pleasure, security, positive feedback from observers). New information might strike an outsider as a basis for rethinking the rationale for a conflict, yet be discounted or reinterpreted by the parties themselves to make it consistent with the prevailing view of the conflict. The greatest potential for a sudden eruption of violence exists when there is a challenge to the validity of a party's attractor.

Conflict attractors are states or patterns that unfold overtime in situations of conflict which resist change or which resume after changes have been initiated. For instance, when people face situations of conflict over a period of time they display certain tendencies to think, feel and react in particular ways. These tendencies can reflect previous life experiences, personality differences, reactions to specific circumstances, or a personal history of conflict with a particular person or group of people. They can be strong tendencies, which overwhelming "attract" the thoughts, feelings, and behaviors involved in conflict, or weak tendencies representing a
passing urge to react in a particular manner. Unlike traditional characterizations of such tendencies as fixed-structures (e.g., competitive personalities), they are dynamic and in a constant state of flux. In fact, people can have several different types of conflict attractors and move between them during the progression of a conflict. If a person experiences conflict when fatigued, for example, he or she might fly-off into a defensive rage and can't help but escalate the situation and make matters worse. However, when the same person is rested, he or she has patience and is able to use humor and creativity to placate most problems. In other words, conflict attractors are the psychological, social, and cultural patterns people and groups display overtime in conflicts, and to which they return after temporary changes occur. Attractors "attract" nearby states, which means that they are central to disambiguating events and enhancing certainty about a course of action. Thus, forces that dislodge a person or group from a way of thinking or acting which is consistent with a conflict attractor may only have a short-term effect due to stronger forces pulling the entity back to the attractor. Some attractors are obvious, because the current state of the system is located within that attractor and thus the associated behaviors are in evidence (such as when a series of battles between disputants provides evidence of a negative conflict attractor). Other attractors are latent, reflecting potential patterns of thinking, feeling, and behaving that are currently invisible but that can ready people and groups to display qualitatively different forms of conflict behavior when instigated.

Attractors are created when the various elements associated with an interpersonal or intergroup relationship become linked and take on a common meaning (e.g., evaluation). A person may be understood with respect to a variety of distinct features, for example, but there is a tendency for these features to be integrated to forge a global judgment of him or her. Similarly, a group may consist of individuals who differ in important respects, but there is a tendency to ignore this variability and develop a judgment of the group as a whole. This collapse in complexity occurs when the elements (e.g., the characteristics within a person, the individuals within a group) become linked by positive as opposed to negative feedback. Positive feedback refers to mutual reinforcement and amplification, whereas negative feedback refers to compensatory relations among elements. Under negative feedback, hostile thoughts and actions toward another person might promote benign thoughts that reign in one’s potential for aggression toward the person. Under positive feedback, however, hostile thoughts and actions recruit other hostile thoughts and actions, promoting both a loss of complexity in one’s orientation and an amplification of one’s hostile tendencies.

In most interpersonal and intergroup contexts, there are many elements of relevant information and many possible ways of configuring these elements to achieve a coherent perspective and a stable platform for action. Just as perceptual elements can be organized to promote different Gestalts, social information can take on a host of different meanings with diverse action possibilities. At any given time, however, only one attractor is likely to be manifest in people’s orientation toward other people or groups — just as a single figure-ground relation characterizes perception at a single point in time. The other possible attractors are latent and may be invisible, both to the parties involved and to outside observers. Yet these latent
attractors may assume prepotence in sudden fashion under some circumstances and promote a notable change in the relations among people and groups. A central challenge for the dynamical perspective is to identify the latent attractors in situations of conflict and investigate the factors that can make them manifest. The hope is that seemingly intractable conflicts can be resolved by activating a latent attractor with benign implications for thought and behavior.

From a dynamical perspective, the key to conflict resolution is to utilize understanding of how attractors are created, so that they can be essentially “reverse engineered.” Attrac tors develop as separate elements (e.g., issues, events, pieces of information) become linked by positive feedback to promote a global perspective and action orientation. Reverse engineering would thus entail changing some of the feedback loops from positive to negative, thereby lowering the level of integration in the system. This is easier said than done, of course.

In one scenario, one could reinstate the salience of individual elements, devoid of their integration with other elements. Psychological research provides clues regarding this “disassembly process.” For example, disruptions to ongoing action tend to make people sensitive to the over-learned details of the action, as do instructions to focus on the details of a narrative rather than focusing on the narrative’s larger meaning. When habitual actions and generalized schemas are deconstructed in this way, people become vulnerable to new interpretations that provide an avenue of emergence to a coherent perspective. In effect, the tack is to recapture the complexity of a conflict attractor and reconfigure the elements to promote a more benign form of coherence.

3 A Specific Mathematical Model: The Dynamics of the Conflict Between Two Actors

Today, the physical sciences use mathematics as the language to describe phenomena in the natural world. Equations are used to encode how we think different physical mechanisms work in the experiments that have been performed. Those equations can also be used to predict what would happen in experiments that have not yet been performed. But, it was not always like this. It is only over the last three hundred years that the physical sciences used mathematics. Earlier, the physical sciences used words to describe nature. What was gained by this change from words to equations?

The physical sciences found that the beautiful brutality of the rigor of mathematics forced people to organize their ideas into clear and often concise statements. It also made it possible to much more easily and accurately determine the logically necessary consequences of their assumptions. Although some people (actually quite a lot of people) find mathematics difficult and intimidating (and even frightening), the value of mathematics is that once a problem is defined it is often easy to use well established methods to "solve" the equations, that is to determine the logical consequences of the assumptions that the original equations represent. It is actually much harder to think in words, with their ambiguous meanings and logical
tangents, than the rigor imposed by the mathematics. Different people can start with the same concepts and reasoning with them, perhaps adding their own most relevant examples or insights, and reach radically different conclusions. With mathematics, the rules of reasoning are rigorously defined, so that everyone starting from the same assumptions must reach the same conclusions.

Today, most social sciences use words as the language to describe phenomena in the natural world. Perhaps here too, using mathematics may provide us a way to: 1) more clearly define our assumptions by forcing us to translate them into unforgivingly unambiguous equations and 2) to help us better determine and therefore understand the logically necessary consequences of our assumptions.

This mathematical approach provides an alternative way to use dynamical systems theory than that used in the metaphorical approach of the attractor paradigm described above. Each approach has its own intrinsic value and limitation. The metaphorical approach gives us new insights by organizing and making sense out of observational and experimental data, but it is limited in that since the rigorous mathematics of dynamical systems is not used, the full strength of those concepts is not being applied. On the other hand, although this specific mathematical model involves a rigorous mathematical application of dynamical systems theory, no one should think that the complexity of human behavior can be fully captured in the solution of a few simple equations. By using both approaches we get to look out of two windows, facing in different directions, to better see the nature of conflicts and their resolution.

It is in this spirit that we present a mathematical model of the conflict between two actors. These actors could be individuals, groups, or nations. We will encode into equations some very simple rules of the social mechanisms of how people interact and then use mathematics to determine the logical consequences of those rules. The intent is not to make a mathematical model of human beings or their behaviors. The goal is simply to use the mathematics as a tool to understand the logically necessary consequences of our assumptions. We do this to understand what each social mechanism does. We can see whether these social mechanisms really do what we think they do, or whether they display surprising effects. In fact, we found that some of our results have matched our intuition (which is good) but other results were surprising and interesting and have given us a different perspective in understanding conflicts.

The variables in the model, x and y, are respectively the emotional state of each actor. The rate of change of the emotional state of each actor (dx/dt and dy/dt) depends on the sum of their “inertia” to change (m1x and m2y, where m1, m2 < 0), the effect of their uninfluenced state when they are alone (b1 and b2), and the influence that each actor exerts on the other. We then calculated how the time evolution of the conflict, as reflected by x(t) and y(t), depends on the interaction between the actors. (This was done by numerically integrating the equations and by an analytical stability analysis to determine the critical points and the nature of their eigenvalues.)

The influence of one actor on another was based on the seminal work of M. Deutsch who demonstrated the importance of the role of cooperation or competition
in the feedback between two parties. As he describes it, “To put it colloquially, if you’re positively linked with another, then you sink or swim together; with negative linkage, if the other sinks, you swim, and if the other swims, you sink” (Deutsch, 2006). In our model, cooperation is modeled as positive feedback between the groups, that is, a positive state of one group increases the positive state of the other group and a negative state of one group increases the negative state of the other group. Competition is modeled as negative feedback, that is, a positive state of one group increases the negative state of the other group and a negative state of one group increases the positive state of the other group. Mathematically, we chose a hyperbolic tangent function to represent this feedback because for this function the influence of one actor on the other is proportional at low emotional states and yet it reaches a limiting threshold at higher levels. The equations of the model are then given by \( \frac{dx}{dt} = m_1x + b_1 + c_1 \tanh(y) \) and \( \frac{dy}{dt} = m_2x + b_2 + c_2 \tanh(x) \), where \( c_1 \) and \( c_2 \) are greater than zero for positive feedback and less than zero for negative feedback. We studied three cases: 1) when there is positive feedback between both actors, 2) when there is negative feedback between both actors, and 3) where one actor responds with positive feedback and the other actor responds with negative feedback. The results of the model are summarized below. The mathematical details are available at http://www.ccs.fau.edu/~liebovitch/NCMR.pdf.

First, we found that for all three cases, when the strength of the feedback is small, that is when it is below a threshold that is quantitatively equal to the “interia” to change, then both actors evolve to a neutral state. It is as if there was no feedback between them at all, even though there is a proportional influence between them. This is an interesting, and unexpected result.

Second, when the strength of the influence between both actors exceeds this threshold, then their behavior changes dramatically. For positive-positive feedback, both evolve towards a positive emotional state or both evolve towards a negative emotional state, they swim or sink together. For negative-negative feedback, one evolves to a positive emotional state and the other to a negative emotional state, one swims and the other sinks. The fact that the results from our model matches the intuitive expectations so well described by Deutsch increases our confidence in the validity and utility of this model. The unexpected result of the existence of the threshold of influence required for this effect interestingly has been observed in social psychological experiments. Another unexpected finding is that which of the two actors swims or sinks depends sensitively on their initial emotional states and on their uninfluenced emotional states.

Third, it is not possible, at least for us, to work out in our own minds or in our words, what to expect for the third case, when one actor responds with positive feedback and the other responds with negative feedback. The mathematical solution of the equations, however, simply leads us to the clear result that in this case the emotional states of both actors oscillate, up and down, until they both slowly relax to a neutral state. The existence of these oscillations illustrates how the mathematics can reveal the logical consequences of our assumptions that we would be very unlikely to discover by simply reasoning with words. This also leads us to an interesting new observation. If the negative feedback between two actors has resulted
in a hurting stalemate with one actor always satisfied and the other always unsatisfied (such as that representative of many national, ethnic, or religious conflicts), then that conflict can be changed to a neutral state by the action of one actor alone switching their strategy from negative to positive feedback. This behavior parallels behaviors observed in real conflicts. If one actor changes his or her behavior from hostile to friendly, without asking for a quid pro quo from the other actor, the other actor may change his or her behavior. (Anecdotally, I find, that in a tense argument, sincerely giving in to my wife may often changes her demeanor and emotional state, leading to a resolution of the conflict.)

Fourth, if there is negative feedback between both actors, and if they both have exactly the same parameters and start with exactly the same initial conditions, both evolve to a neutral state. But, if they have ever so slightly different parameters or start with ever so slightly different initial conditions, they first tend to a neutral state, but then one evolves to a highly positive emotional state and the other to a highly negative emotional state. What is fascinating, and completely unexpected, is that they first approach this neutral state, before so sharply diverging to their final very different states. (Technically, this neutral state is called a "saddle", because the values of the variables in time are each drawn toward it in one direction and separate from it, and from each, other along a different direction, while the final states are called "fixed points" because the values of the variables in time are drawn to them from all directions.) Observing the emotional states of the actors sharply diverging from each other, one might conclude that at that moment there was a precipitating event that generated that divergence. In fact, that is not at all the case in this model. It is the small difference in the parameters or initial conditions that first drew the actors together and then separated them. Again, the mathematical analysis reveals behaviors to us that are the logically necessary consequence of our assumptions that we would be very unlikely to be able to reach by reasoning with words alone.

Fifth, this model also gives us insights into how actors might change their strategies in a conflict to benefit themselves. As we have already seen, the positive-positive or negative-negative feedback cases reach a final steady state. However, one actor alone, by switching his or her strategy, can change the system into the mixed positive-negative feedback case. As explained above, in this case the emotional state of both actors oscillate, each passing through positive and negative values at different times. If that one actor now switches back to his or her original feedback strategy, it is equivalent to running the original positive-positive or negative-negative feedback case with the initial conditions present at the moment the actor switches back. Since the positive-positive or negative-negative cases are sensitive to these initial conditions, this means that the roles of winner and loser can be reversed. We have demonstrated this behavior in the numerical integrations of these equations. Since we can use the mathematics to determine how these changes depend on the parameters of the model, we can therefore make the following two intriguing predictions: 1) the more the inertia to change, the more often an actor can gain an advantage by switching strategies, and 2) the stronger the feedback between the actors, the shorter should be the duration of his or her switch to gain this advantage.
Sixth, we have now begun to explore the behavior of a network of actors linked by either positive or negative feedback. Even at this very stage of exploration, we have already found some very intriguing results. For example, if all the actors respond with negative feedback, the network will spontaneously break into two groups. The actors in each group all share the same emotional state (positive or negative), and the two groups will have opposite emotional states. Such phenomena may shed light on how a seemingly stable social environment can suddenly degenerate into two warring factions. We are also studying networks where the actors are linked by multiple types of influences, to understand under what circumstances these different elements of social interaction collapse together so that all these different elements are tainted by the conflict.

The fact that some features of this mathematical model match previous experimental observations and our own intuition about conflicts gives us some confidence in the validity and the utility of the model. The model as well gives us new insights to explore in understanding the effects of the positive or negative feedback. It also challenges us to figure out how to analyze the field or experimental data on conflicts to determine the values of the parameters of the model.

**Bibliography**


