Wavelike Design of Social Agents Simulated as System of Interacting Net of Neural Networks

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Instead of following particles like interaction scheme between agents in the social domain, we have abandoned mechanistic and proposed conceptually wavelike approach. In fact, we assumed that the major modern capital accumulation is in the form of knowledge (information and environmental resources), which, being dimensionless according to information theory, has some similarities with energy. Therefore, we constructed universal rules to represent agent's various wealth and behavioral patterns in the form of wave packets composing an agent's unique energy spectrum. In practical terms, proposed model rests on the net of simple backpropagation NN, which are designed for simulation of agents' energy spectra, where each NN receives, transforms and transmits the spectra bands, following initial training on *a priori* settled rules. Hence, our simulated economic system rests on the economic model, which is based on the set of horizontal and vertical production rules used appropriately for NN training. The simulation model is also linked with some practical aspects targeted for empirical validation and model realization.

1. Introduction

By getting inspired from natural systems, social scientists and engineers are starting to understand that, to construct self-organizing and adaptive systems, it may be more appropriate focusing on the engineering of the proper interaction mechanisms for the components of the system, rather than on the engineering of their overall system behavior [Mamei 2006].

In line with above consideration, this article focuses on a physically inspired interaction model, i.e., field-based coordinated multiagent system, which relies on virtual computational fields, mimicking gravitational and electromagnetic fields, as the basic mechanisms with which to coordinate activities in open and dynamic ensembles of social agents.

The study of coordinated models goes beyond computer science, in that also evolutionary computation, behavioral sciences, social sciences, business management, artificial intelligence, and logistics somewhat strictly deal with how social agents can properly coordinate with each other and emerge as globally coherent behaviors from local interactions [Mamei 2006, Malone 1994, Bar–Yam 1997].

According to Tesfatsion, economies are complex dynamic systems, where large numbers of micro agents engage repeatedly in local interactions, giving rise to global regularities which in turn feed back into the determination of local interactions. The result is an intricate system of interdependent feedback loops connecting micro behaviors, interaction patterns, and global regularities. As elaborated by numerous commentators, the modeler must now come to grips with challenging issues such as asymmetric information, strategic interaction, expectation formation on the basis of limited information, mutual learning, social norms, transaction costs, externalities, market power, predation, collusion etc.

Uncoupled and indirect interactions among agents require the capability of affecting the context and of perceiving it. The context is modeled here as virtual data fields, where each spatial or logical node stores the pervasive field values (it is possible to define a field in analytical terms, by writing equations defining its values in space). The model promotes mediated interactions by exploiting some sort of distributed information that can be used as a means to enforce indirect and uncoupled interactions among agents and that can also be expressive enough to represent contextual information in a form locally accessible and immediately usable by agents.

One of the closest examples in this area is amorphous computing [Nagpal 2004]. Another interesting proposal in that direction is the Multilayered Multi Agent Situated System (MMASS), defining a formal and computational framework relying on a layered environmental abstraction [De Paoli 2003]. MMASS were related to the simulation of artificial societies and social phenomena, for which the physical layers of the environment were also virtual spatial abstractions. In the last decade, a number of other field-based approaches were introduced like Gradient Routing (GRAd), Directed Diffusion, "Co-Fields" at TOTA Programming Model, CONRO etc [Mamei 2006].

In fact, almost all proposed systems are either employed for various technological or robotics applications and very few of them like MMASS, Agent-Based Computational Demography (ABCD) or Agent-Based Computational Economics (ACE Trading World application: simulation of economic phenomena as complex dynamic systems using large numbers of economic agents involved in distributed local interactions [Testafsion 2006]) are suitable for programmable simulations of social phenomena (including economic macro modeling and financial economics, which is of particular interest in this paper).

During the 1990s, computational approach to the study of human behavior developed through a vast quantity of literature. These include approaches that range from the so-called evolutionary computation (genetic algorithms and evolution of groups of rules) to the study of the social evolution of adaptive behaviors, of learning, of innovation, or of the possible social interactions connected to the theory of games [Billari 2006].

More specifically, the current paper addresses agent-based computational finance. An excellent overview of early and influential models in agent-based computational finance is given by Lebaron [Lebaron 2005]. The models range from relatively simple to very complicated like the Santa Fe Artificial Stock Market [Hoffman 2007]. However, unlike the literature on agent based computational finance in general, the literature on agent-based computational behavioral finance is still scarce [Takahashi 2003]. For the present day we have just a few working examples for the investment finance like SimStockExchange [Hoffman 2007], U-Mart System (see Proceedings of the First World Congress on Social Simulation, WCSS'06, Kyoto University, Japan).

Hence, this interdisciplinary paper examines accordingly how to (i) simulate dynamic self-organizing complex social behavior using a novel type of field-based approach, (ii) enhance the simulated agents' intelligence using energy spectra for the NN-based approach.

In fact, our model being universal is specifically tailored here for the simulation of investment markets. This research aims to construct a simple neural network (NN) based multiagent system of heterogeneous agents' targeted to get on the efficiency frontier by combining investments to the real life index funds and nonrisky financial assets, diversifying the risk and maximizing the profits. Each agent is represented by the different stock trading strategy according to his portfolio, saving and risk aversion preferences. The goal is, following empirical evidences from the real investment markets (Pareto wealth, Levy returns distribution [Blume 2006] etc), to find enough proofs that NN-based multiagent system, in principle, has the same fundamental properties of real investment markets and can be used further to simulate even more complex social phenomena.

The current interdisciplinary approach is driven by the author's deep driving desire to go beyond mechanistic Newtonian style understanding of social phenomena. In some sense, proposed ideas might lead us from material realism towards new metaphysics of monistic idealism, which posits unitary philosophy (unity in diversity). As worldwide known particle physics and quantum theory experts like Erwin Schrodinger, Fritjof Capra, Amit Goswami, Alan Wolf and many others have articulated "Consciousness is a singular for which there is no plural" meaning that transcending consciousness creates both material and mental worlds [Goswami 1995].

In the simplest sense, the proposed model distantly adopts this thinking as it (i) has all (material and informational) objects interpreted as compositions of waves, which are coded in the universal ladder of energy, i.e. spectra, (ii) uses production rules, which are incorporated in the same spectra, for transformation between various resources. To our knowledge, there is no experiment in the field that focuses on the market mechanisms in such a virtual setting.

The remainder of the paper is organized as follows: Section 2 briefly discusses the model setup and constituent components. Section 3 subsequently describes novel wavelike experimental design. Section 4 gives a short glimpse over NN-based multiagent simulation and section 5 describes internal functionality of an agent, i.e. production rules component. Section 6 finally concludes the paper.

2. Model Setup

As a number of theoretical and empirical studies have suggested, the proposed investment simulation model has micro (for each investing agent) and macro (for the whole system of agents) goals. The micro goals are to maximize disposable capital and survive in the long run. The macro goals are to create self-organized social system capable: 1) to oscillate in a periodic fashion, following known economic cycles, 2) to oscillate in a nonperiodic fashion, following statistical cycles (R/S analyses), 3) to generate clusters in time and space , 4) to generate fractal invariants following known estimates for the real markets.

To begin with, let's start from the simple general UML scheme, which is presented in Figure 1. It depicts main components of the proposed model and relates them into the modular scheme (modularity secures flexibility, adaptability and self-organization). The model consists of five main components: 1) resources (information and natural resources), 2) wavelike transformation (WT), 3) neural networks (NN), 4) evolutionary computing (EC), 5) production rules (PR).

Simulation component links all other components and resources into a single software package, which controls the execution of simulation, change and monitoring of parameters and solution space via GUI. The limited space available, though, doesn't allow presenting detailed description of components in this paper. Therefore, below there are given only short comments, whereas, more detailed description is available in the paper [Plikynas 2007].

Resources Component (see Fig. 1). This component is intended for simulation and management of spatially distributed information and natural resources as valuable capital resources vitally needed for agents survival. Both, information and natural resources are converted to the energy equivalent represented by different spectra, where agents are capable to transform and use spectra for maintaining their operations. In general, information resources are 1) behavioral rules, 2) internal information generated by NN, 3) external (initial and run-time) information generated by the user.

Wavelike Transformation (WT) Component. This component is intended for preprocessing, adapting and transforming for each node on the grid locally available information and environment (natural) resources to wavelike spectra, where various compositions of spectrum bands represent different resources, rules, information, capital available, energy etc. Initially trained NN are used for interpretation of spectra and generation of responses (other spectra).

Neural Network (NN) Component. Each agent is a modular entity composed by two layers of NN, which are dealing with a different market data at the first layer and with operational decisions at the second layer. Whereas, management and monitoring of the multiagent system is conducted with the algorithmic shell of procedures. NN are initially trained to forecast, recognize and simulate (e.g. investing strategies, timing, behavioral rules, resources etc).

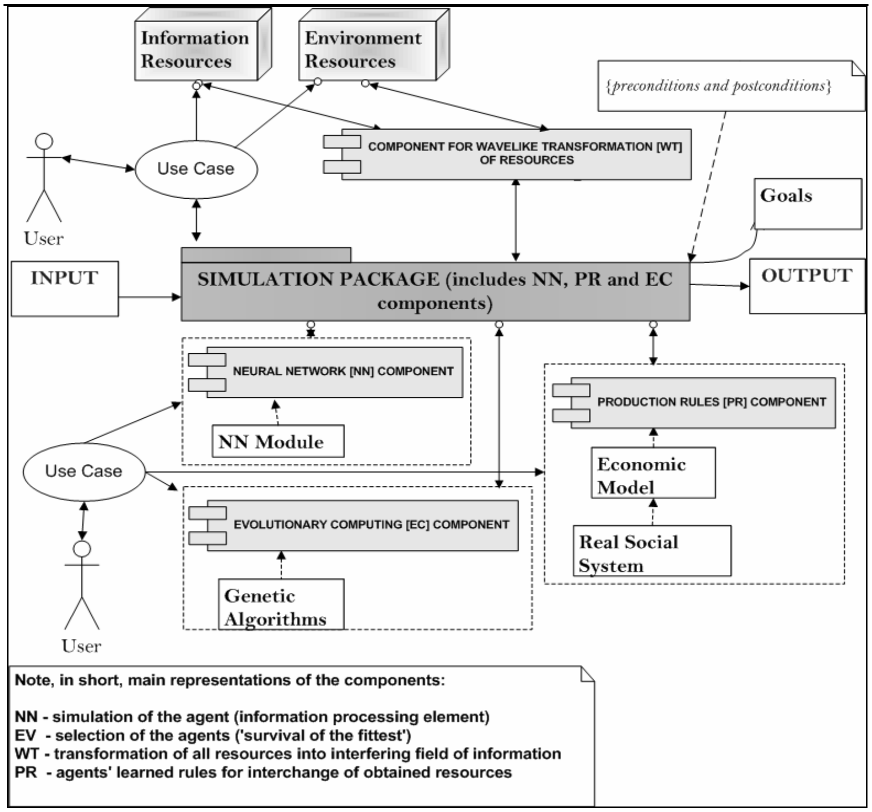

Figure 1. Model setup: UML deployment scheme

Evolutionary Computing (EC) Component. The major goal is selecting those agents who are doing better job according to the chosen criteria (capital accumulated, proximity to the resources etc) and crossover them with each other for production of the new offsprings. The controlled rate of crossovers and random mutations are used for qualitatively unique shifts at the next offsprings' generation. This component is controlling the adaptiveness of the whole system and individual agents' as well. The chromosomes here are composed by the agents' internal parameters like NN configurations, weight matrixes, investment strategies used, behavioral rules etc.

Production Rules (PR) Component. This is an algorithmic shell for managing behavioral rules, which, being agent's internal resource, are used for description of how one resource is transformed to another, e.g. how available capital (interpreted as energy) is transformed to consumption, displacement (x,y), production, creation of new rules etc. In general, there are three types of behavioral rules for transformation of (i) information and capital resources, (ii) technological resources, (iii) innovation resources.

3. Wavelike Experimental Design

The empirical study described below seeks to create self-organizing system of investing agents. Following well-established definition [Wooldridge 2002], investors simulated as NN-based economic agents have properties of distributed/concurrent systems resembling computational information processing entities, i.e. computational agents. In fact, we are modeling here a reactive system that cannot adequately be described by the relational or functional approach.

Complexity of the new approach suggested not only modularity but also multiple phases for achieving the goals. The first faze was meant to create a simple NN-based agent, the second – the most simple system of agents (agents do not communicate with each other), and the third - multiagent system where agents have functionality mentioned in the previous section. Currently we are experimenting in the second phase, but the scope of this paper is too small to reveal experimental results. Therefore, this paper concentrates more about methodic and conceptual parts.

Each investor is represented as agent, where total number of agents varies from 100 to 1000. Agents technically are modeled using multilayer perceptron [MLP] backpropagation neural networks [NN] with the fast Levenberg-Marquardt [LM] learning algorithm on the Matlab NN Toolbox platform. All real market data is preprocessed (missing values eliminated, chronology adjusted etc), but not detrended as we want all periodic and nonperiodic fluctuations be present for NNs to learn. As a standard procedure, each NN goes through the initial training and validation stage. Only then they are ready for the simulation stage, i.e. forecasting of trends and recognition of investment strategies. In other words agent learns to forecast investment market behavior (index fluctuations, recognize specific investment patterns) and take appropriate decisions (market intervention timing moments, and own resources allocation). An agent is given opportunity (depending on his trading strategy) to allocate his capital to different mutual funds based on the popular indices like S&P500, Dow Jones Composite or invest to nonrisky US Treasury Bonds.

It is impossible to cover the full range of our work here, but to give a flavor of it the major components (see Fig. 1) of the model are somewhat uncovered below, see Fig. 2.

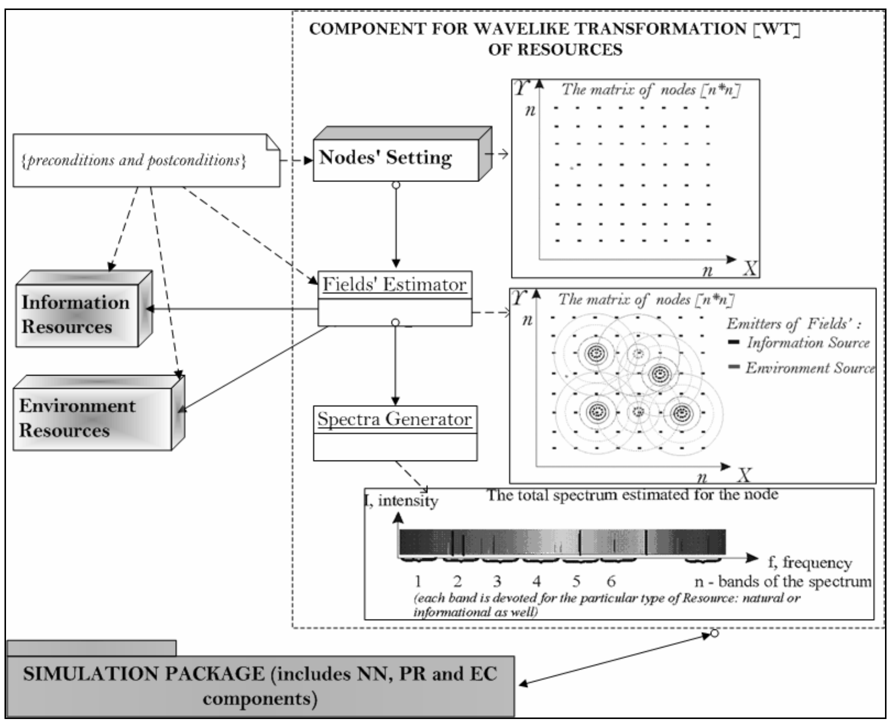

Figure 2. Component for wavelike transformation of resources.

For the effective implementation of spectra as universal energy-information warehouse, we first have to transform all tangible objects-resources to the energy equivalents and then to interrelate different types of energy as intangible information stored in the form of corresponding sets of spectral bands (see Fig. 2). The deep meaning of it is based on the principle of reductionism and universality as we are looking for the most universal means to reduce multiplicity of forms into the singularity of content.

Notwithstanding the theoretical flexibility of the model, the practical implementation is not as trivial as it looks like. We may obviously do so by actually finding those relations, however, such an approach is often difficult if not impossible to nail down for the real systems. Therefore, the current research is dealing with the few simplifications.

First of all, the model deals with discontinues rectangular grid of spatial nodes, where natural resources and agents are distributed. For the larger grids (over 100*100 nodes) field values are calculated only for the nonempty nodes, i.e. those nodes where we have an agent or we are interested to know the field intensities.

The NN-based software module is attuned to algorithmically calculate transmission and superposition of waves' frequency, phase and intensity for corresponding nodes. The sources of waves are not only environmental (natural) resources (which are initially distributed in random way) but also agents, who transmit some information about their capital, location, behavioral patterns they posses etc. Fields' intensity decreases according to the $1/R^2$ law.

The major premise holds that all resources are in the form of potential energy, e.g. like electric field, which diminishes as inverse ratio of square power of the distance. So natural resources emanate it unwillingly, but internal information resources can be emanated at will according to the chosen strategies by particular agent. Meanwhile, propagation of fields and their damping dawn are calculated in a similar way as for physical systems. For instance, for the environmental (natural) resources (there different type of natural resources like raw materials, food etc) fields' intensities are emitted proportionally to the amount of the natural resource, where frequencies depend on the type of resource. Whereas, for the agent's internal information resources waves are emitted with other frequencies depending on the sort of information an agent posses and is willing to transmit. The duration of waves emission is limited for the settled damping period (in time, if not reactivated, source intensity diminishes until emition ceases).

So, Fields' Estimator (see Fig. 2) calculates the aggregates at each node for various waves coming from different resources. This module produces spectra for the nodes. Each spectrum zone is devoted for the particular type of resource: natural or informational. In qualitative terms different resources have their own dedicated spectrum zones where each resource has a unique composition of spectrum bands (waves). In this way all agent's resources, including even location (X, Y), have their own separate spectral zones, where they are appropriately represented. For instance, if two energy sources are located at (x1, y1) & (x2, y2) with frequencies and intensities as (f1, I1) & (f2, I2) accordingly, then there are certain ways to reflect them both on the single spectrum as separate bands and calculate their superposition [Mamei 2006, Malone 1994].

4. NN-Based Multiagent Simulation

Notwithstanding the theoretical flexibility of the wavelike approach, our model is still lacking description of the major building block, i.e. NN, which has responsibility to implement the wavelike functionality. Some very brief comments are given below, see Fig. 3.

According to the above discussion, using NN notation we mean a neural network as backpropagation multilayer perceptron, which can be initially trained (for performance of certain behavioral rules represented in our case by different investment strategies). Meanwhile, NNN (net of neural networks) notation is used for the set of NN which represent a system of agents.

NN inputs are multidimensional numerical data (with feedback loops and autoregressions) in the first NN layer. However, NN inputs are not physically connected to the other neural networks. This is important difference comparing with other methods. Instead, they are connected to the spectrum zones generated at the local node, see Fig. 3 (the first picture from the bottom). At the node all waves coming from other resources (natural or informational) are superposed using WT component (see Fig. 2).

In the second NN layer, inputs are derived from the first NN layer and also from the internal states (e.g. as retained memory from the previous states), but everything is in the form of spectra. Consequently, spectra are used as the mean where all resources, energy and information are settled in. In some sense, spectra depict a screen shot of the potential fields, which pervade through the medium (potential fields can be interpreted as energy-information spreading through the medium).

One of the most distinguished and novel features of the proposed approach are related with how we are handling wavelike representations of resources and time dependant market functions. In fact, the former are represented as single spectral lines (bands) in the appropriate frequency zones and latter as compositions of several bands. All time dependant functions are transformed to the frequency domain using Fourier spectra (Fourier synthesis is used to get back to time domain again). In operational sense it helps to eliminate time factor shifting to frequency domain, which is dealing with energy and information in a very compacted and efficient way [Benenson 2002].

MatLab NN Toolbox is used for the implementation of the NN functionality (see Fig. 3). During the training phase a NN (or agent) learns how to 'live' in the chaotic environment following some targets, i.e. to maximize own energy (capital) and to learn some production rules. NN are trained for different investing strategies following technical trading indicators like MACD, MFI, WMA, W%R [Plikynas 2006, Jones 2002]. In this way, we generate a set of different agents.

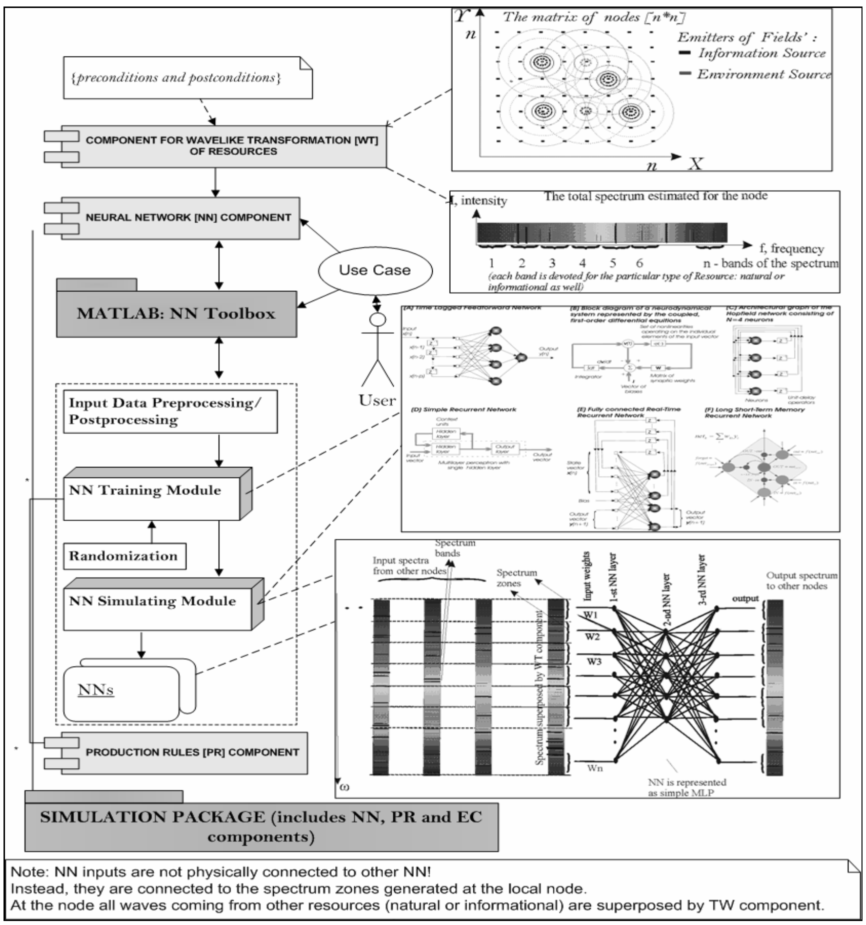

Figure 3. Neural network component

However, the degree and depth to which net of neural networks (NNN) is performing in evolutionary sense depends on the NNN architecture. We have to use some global selection criteria to make it evolve as dynamic process. Therefore, the author proposes to use genetic algorithms, which employ selection, mutation and crossover procedures [Haupt 2004] for ensuring survival of the fittest agents according to the global selection criteria. Evolutionary computation (EC) module (see Fig. 1) does just that.

The adopted selection-mutation equation, also called Fisher-Eigen equation, see [Tesfatsion 2006], describes mutation and replication dynamics, where mutation matrix $\omega(a'|a'',t)$ has to be chosen in advance. In the economic context, it can be chosen according to the probabilities of individuals randomly switching from one choice to another. It is even possible to make this matrix dependent on the actual situation (such as the situation of the individuals or their knowledge about potential improvements).

The utility or payoff $u\,(a,t)$ is calculated for each action a on the basis of a finite memory. However, the action that caused the highest payoffs in the past is not immediately chosen. Instead, the behavior converges towards the choice of the action with the best experience. Such a model seems to be a good choice for modeling a realistic combination of experience collection and experimentation. In short, a very simple model that combines the routine-based processes of experimentation, experience collection and imitation/communication is described by Thomas Brenner, see [Tesfatsion 2006]. Besides, there plenty of other employable models like EWA model which uses two fundamental types of learning processes: reinforcement learning and believe leaning. This model is designed such that it describes these two learning processes as border cases for specific choices of the model parameters.

A complete description of the evolutionary modeling, though, would take too much space. Therefore, in the next section we are finalizing with the production rules (PR) component, which is aimed to describe micro level internal functionality of the agent.

5. Production Rules Component

It is impossible to cover a full range of production rules (PR) here, but to give a flavor of it a few details of PR approach are included. First, the author proposes a new approach, i.e. the “Spectral Ladder” (SL) as potential energy equivalent for inclusion of all resources on one universal measuring system, see Fig. 4. Following this approach, more intelligence (in the sense of information quality) resource has, the higher it is positioned on the SL.

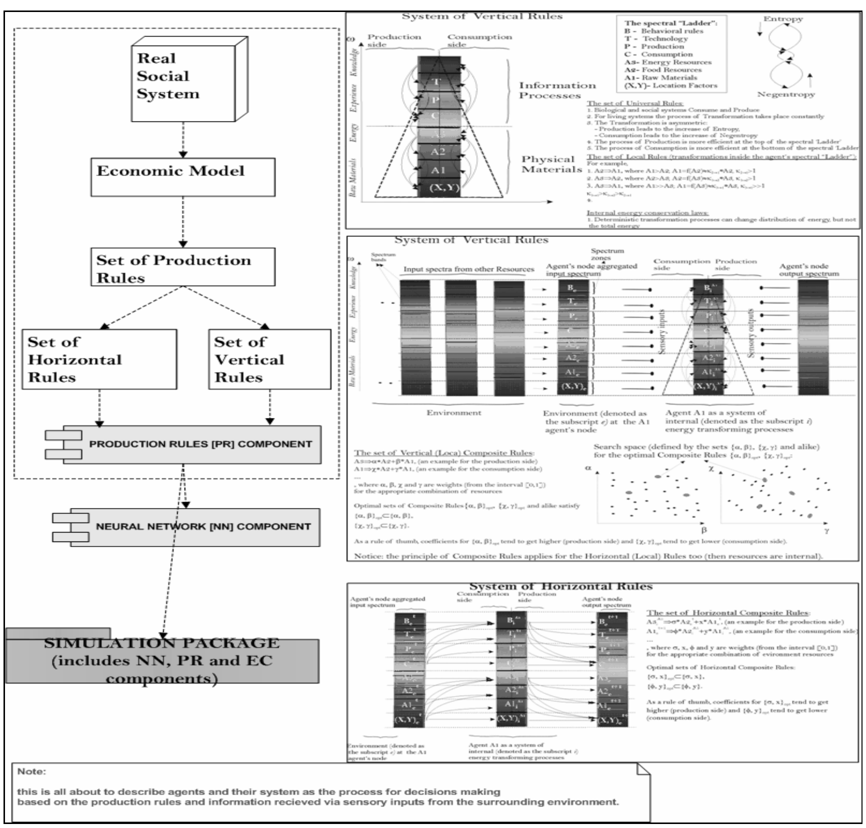

Figure 4. Production rules component

So, resources on the SL are sorted out from the high frequencies to the lower frequencies according to this order: behavioral rules (strategies), technology (a set of agent's individual characteristic parameters), production rules (functions for transforming higher resources to lower on the SL), consumption (functions for transforming lower resources to higher on the SL), kinetic energy, food, raw materials. In general, production rules describe how agent's resources and their derivatives are functionally interlinked in the system of equations, inequalities and constraints.

The author introduces two sets of rules governing agent's behavior, i.e. vertical rules (VR) and horizontal rules (HR), see Fig. 4. VR are briefly described above: it is all about how agent's internal resources are qualitatively and quantitatively transformed as if one sort of potential inner energy is transformed to another following some sort of energy conservation laws, see SL. Meanwhile, HR are aimed to describe how resources are "traded" between different agents in the close neighborhood (there are not only spatial but also virtual spaces, see [Mamei 2006]).

In reality, proposed approach is taking use not only from most fundamental laws of physics but also from not less important information laws like entropy and negentropy, which govern the interplay between self-organized order, chaos and white noise [Arndt 2004]. The scope of information-based approach is so vast that the author would need another paper to describe it.

In the proposed social system, at the local scale (micro level) agents are following individual strategies as they are endogenously bounded to the personalized genetic code, which shapes theirs identities, e.g. goal vectors (competing for resources), marginal propensity to consume/invest, personal budgets (allocation of disposable incomes), perceived utilities, accepted welfare vectors etc. System of such agents exogenously forms the macro scale phenomena like total budget, investing rates, entry barriers, clusters, economic periodic and nonperiodic cycles, herd-behavior etc.

The particular set of production rules is obtained from the Economic Model (EM), which depicts major behavioral patterns of the simulated real social system.

6. Discussion and Conclusions

While an incentive-based model would rather focus on facts and phenomena, external to the agent, a knowledge-based view has to thoroughly investigate the agent. In principal, the former – that is orthodox methodology – uses the Newtonian mechanics which requires the concept of a *homo economicus* as a nenecessary condition [Billari 2006]. As much as orthodox methodology asks for such a perfectly rational and therefore homogenous agent within a supposedly deterministic world, the need to shed some light on the non-deterministic aspects – the heterogeneity in agents' behavior – asks for an adequate methodology.

Presented interdisciplinary work is based on some novel theoretical and practical methods and models which potentially are able to shift the commonplace paradigm of Newtonian mechanics to the wavelike reality of fields, energies and information in social domain. The coherent interplay between five major components (R, WT, NN, EC, PR) of the proposed paradigm shifts us from the social multiplicity of forms and representations to the universal world of energy and information. For instance, proposed model of the Spectral Ladder (SL) and associated sets of Vertical and Horizontal rules (VR & HR) are implemented in production rules (PR) component to facilitate the resource-energy-information transformation process.

Following wavelike transformation (WT) component, implementation of spectra as universal coding system for agent's internal and external information gives unique genetic code for each agent. This code can be easily interpreted for selection and mutation mechanisms implemented in the evolutionary computing (EC) component. This paper is also addressing question how to build the basic intelligent block (see neural network (NN) component) for such a system and how to make a net of such blocks capable to process and store the information.

As experimental data shows, the massively parallel and local interactions give rise to path dependencies, dynamic returns and interaction. In such an environment global phenomena such as the development and diffusion of technologies (behavioral rules), the emergence of clusters and networks, herd-behavior etc. which cause the transformation of the observed system can be modeled adequately. The proposed computational laboratory is meant for exploring various market arrangements, potential paths of development. So as to assist and guide e.g. investment firms, policy makers etc. in their particular decision context.

The author is at pain to emphasize that these findings represent an initial first 'take' on simulation of complex social systems. This work, though, gives some clear outlines and their explanatory sources. Moreover, the author has spent a considerable amount of this paper examining the methodological novelty of proposed approaches. Subsequent papers will focus more on the empirical aspects of the current research, which already show feasibility and validity of the presented ideas.

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