Chapter 1

Complex Stellar Dynamics using a hierarchical multi-agent model

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abstract : This paper defines a new approach for cosmological simulation based on complex systems theory : a hierarchical multi-agent system is used to study stellar dynamics. At each level of the model, global behavior emerges from agent interactions. The presented model uses physicaly-based laws and agent-interactions to present stellar structures has the result of self-organisation. Nevertheless a strong bond with cosmology is kept by showing the capacity of the model to exibit structures close to thoses of the observable universe.
keywords : complexity, hierarchical multi-agent system, cosmology, patterns formation

1.1 Introduction

From globular clusters to spiral galaxies, cosmological evolution shows a wide variety of patterns and complex behaviors. For years, numerical simulation in cosmology has tried to reproduce and explain these behaviors by using strictly reductionnist models (such as [10] [14]). Even if this approach has carried out to successes, some problems remain unsolved :

- Observed dynamics highly depend on the number of point-mass particles used in simulation [2][17](strict application of physical laws requires the use of point-mass particles in simulation models). Using this approach, some complex patterns do not appear in the cosmological models till a high number of particles (over 1024^3).

- In addition, a realistic number of point-mass particles should be around 10^{41} [12] for a typical spiral galaxy what is a calculative impossibility.

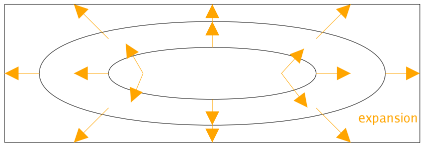
The current approach to solve these problems is to define increasingly precise models taking in account more and more parameters [11] : each physical phenomenon is calculated by a dedicated algorithm (such as gravity [4] [3]) and the results are combined according to the goal of the experiment. The average number of particles currently used is between 128^3 and 512^3.
In the same time, the study of complex systems obtained a certain success, in particular by the use of models like cellular automata [13] [15] [16] . Even if results get by theses models are structurally close to the observable, they are often too abstract and too far away from physical reality to be easily accepted by the cosmologists community.
Therefore, our approach [5] takes advantages of a hierarchical multi-agent model while keeping a strong bond with physics : each complexity level of the model uses physical laws and interactions on a number of agents lower than what is necessary for earlier models (from a calculative point of view) and the various agents at a given level are agregated to form structures of higher level.
In this paper, we will first give an overview of the model, then we will define it more formally. In a second part, one of the main experiments used to validate this model will be shown.

1.2 Hierarchical multi-agent Model

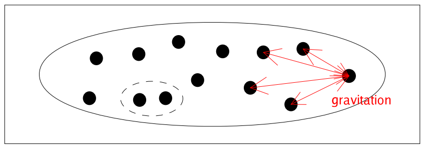
The obvious complexity of the problem and wide variety of forces and scales [11] led us to use of a hierarchical multi-agent system. The aim is to define a high-level model, highly parametrizable to get a picture of universe and to study how cosmological patterns can emerge from interaction within and between complexity levels.

1.2.1 Overview

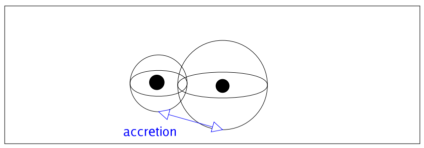
The model is composed by four complexity levels :



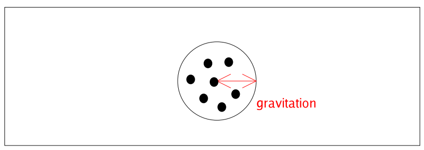
Environnemental actions : all the forces which apply to the whole agents in the universe, such as expansion for instance.



Long range interactions : forces which apply without any distance constraints such as gravity.



Local interactions : interactions applying on a short distance (approximated as a vicinity) such as accretion.



Internal interactions : abstraction of mass matter subjected to gravitational attraction.

1.2.2 Levels Details

Level 1 :

Level one agent are spherical clusters of matter in disordered rotation whose radius and spin vary according to time. We have conducted several experiments to study the behavior of point-mass particles under the effect of gravity, in various conditions of mass, distribution, initial velocity, etc.
Fig.1.1 shows a typical result of one of these experiments.

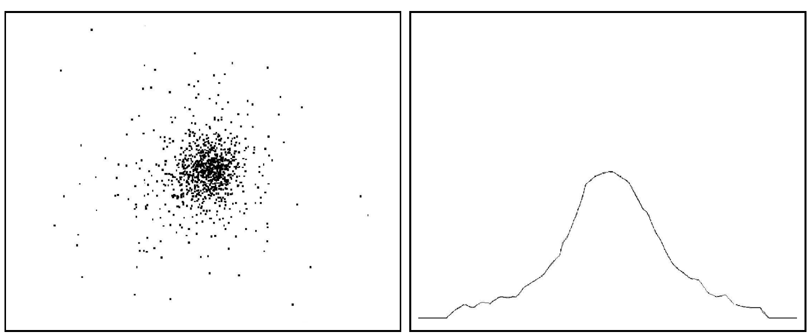


Figure 1.1: *Left : a typical stable structure formed (1 000 particles) by gravity (G fixed at his physical value) application on a random distribution. Right : evolution of the collapse time according to the mass/number of particles ratio*

The structure formed at the end of all these experiments is spherical and corresponds to what we defined as the agents.
To calculate the evolution of the radius, a gravity force will be applied to an element at the periphery of the agent by using this equation :

$$F = G \cdot \frac{m}{r^2}$$

m the global mass of the agent

r the radius of the agent.

An algorithm, widely used in cosmology, will be used as a basis for the gravity application : Treecode [4]. By correlating the experiments we can realize that it exists in all simulations a transitory series of state (limited in the time or not) during which the particles collapse on themselves. The duration and the speed of collapse (from stabilization to total collapsing) are function of the relationship between the overall mass of the system and the number of agents on which it is distributed (Fig.1.1). These results correspond to the evolution function of the radius applied on peripheral elements : agents are approximated as a sphere whose radius and spin varies according to the inner. They can represent a mass of gaz, purely gravitationnal matter (called dark matter) or a star.

Level 2 :

Level two gather all the interactions applying on a short distance such as accretion [9]. Therefore the function U applied by this level of complexity will be :

$$\delta V x_i = \sum_{j=1}^{M} \left[\mu_j^t \cdot \left(V_j^t(x) + s_j^t(x) \right) \right]$$

s_i^t : the spin of the agent i at time t

μ_j^t : the accretion capacity of the agent j at time t

Fig.1.2 shows examples of this concept of vicinity and its physical mapping : structures, self-formed by the effect of gravity, are surrounded of a less dense matter halo on which local forces, such as accretion power, apply.

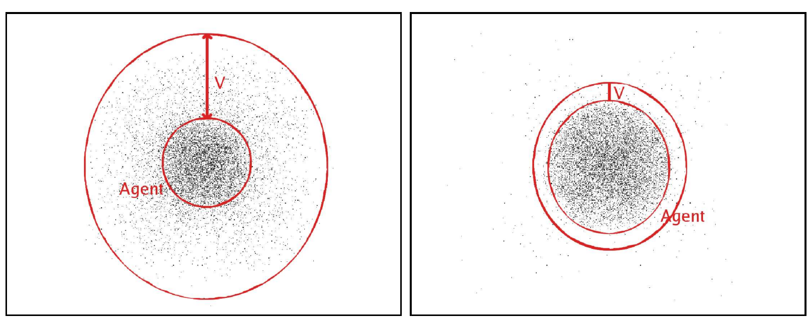


Figure 1.2: *Illustration of the concept of agent and vicinity (v) in the model, from an experiment carried out on a TreeCode.*

Level 3 :

Regarded as the principal force responsible for cosmological structures formation, gravity applies to the whole mass elements present in the universe. It is described by the equation :

$$\frac{dx_i}{dt^2} = G \cdot \sum_{i,j=1 \ / \ i \neq j}^{M} \left(\frac{m_j(x_j - x_i)}{(d_{ij})^3} \right)$$

M : number of agents in the universe
G : gravitational constant
m_i : mass of the agent i
d_{ij} : ragen between agent i and agent j

Level 4 :

The universe, as we know it, is expanding. This can be simulated as a force applied to all the agents and aiming at moving away all elements of the system. The expansion will be set as a radial force applied by the environment to all agents, approximating the formula :

$$v = H \cdot r$$

H Hubble's constant
v velocity of an unspecified point of the universe

1.3 Model Validation

In order to validate the model defined above, we show that the emergent structures are coherent with the observable ones. In this experiment we simulate the collision of two particular galaxies (G1 and G2) and compare it with other simulation and real pictures from spatial telescop Hubble. The structure, called "The Mice", is often studied in cosmology [6] [7] [8].
The experiment is undertaken with the following parameters [6]:

- $mass\ (G1) = 3.95 \cdot 10^{41}$ kg / $mass\ (G2) = 4.05 \cdot 10^{41}$ kg
- $radius\ (G1) = 9.40$ kpc[1] / $radius\ (G2) = 11.0$ kpc
- $1s$ "*simulation time*″ $= 1.10^{14}s$ "*cosmological time*″

The observational data indicate that the collision took place 160 Myr[2] ago. To get the image of simulation corresponding to the current state of these galaxies, it is necessary to let evolve simulation during 220 Myr (which is equivalent to 69s in simulation time with our system).
The distribution of the elements is as follows : 50% of gas (represented in yellow), viscous and subjected to gravitation forces, and 50% of dark matter (represented in white), only affected by gravity.
Fig.1.3 shows the result of this evolution and a comparison with other data.

[1] $1kpc = 3.08568025 \cdot 10^{19}$ m
[2] 1 Myr = 1 million years = $3.1536.10^{13}$s

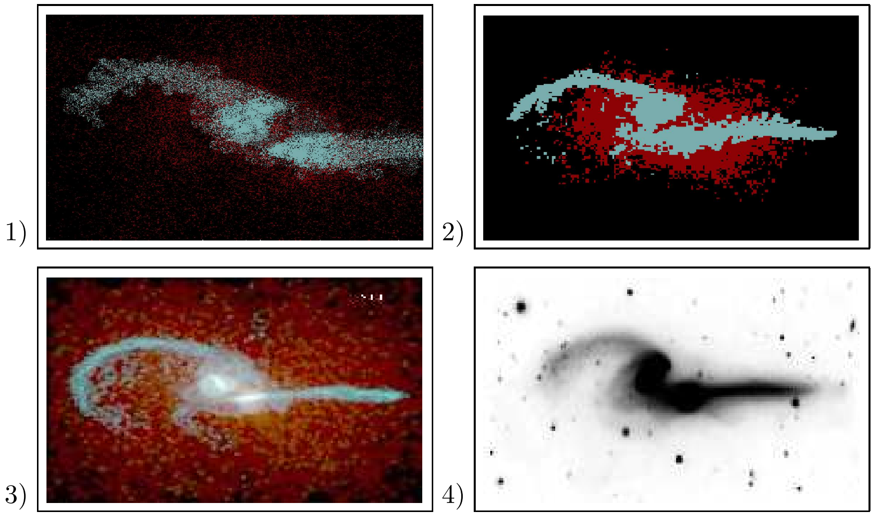


Figure 1.3: *1) and 2) respectively show the evolution of the model with 300 000 agent and 3 000 agents. 3) shows the result of a simulation using dedicated cosmological algorithm by J. Hibbard. 4) shows a picture get by the spatial telescop Hubble (http://hubblesite.org/).*

These results are obtained after the same evolution time and show that the dynamics and the resulting emergent patterns are the same.
To check if our model is less dependent of the number of elements used than a classical model, a series of experiments aiming at repeating the same simulation with a decreasing the number of agents/particles has been carried out. Results show that, in contrast with classical cosmological models, beyond a threshold, the number of elements does not influence dynamics any more.
Fig.1.4 shows a example of this behavior difference.

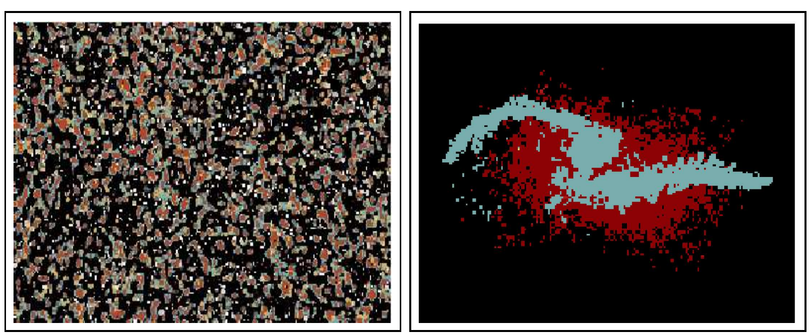


Figure 1.4: *Left, the result of a simulation using dedicated cosmological algorithm (TreeCode) with 3 000 particles. Right, the same simulation time with our model (still 3 000 agents). Dark mater in red. Gas in yellow*

In contrast to the physical models where a reduction of the number of particles leads to a less homogeneous application of forces, the evolution of the agents (and so, interactions) compensate this drift. In a less dense medium the vicinity of the agents increases, modifying the importance of local forces. These forces increase the total cohesion between agents, improving formation and survival of complex structures.
However, under a threshold, the evolution of agents leads to the appearance of vicinity of the same size as the system, which transforms the local forces into global forces and destroy dynamics. This threshold is determined by the size of the universe, the density of matter, etc. The conditions of the experiment above fixes a threshold at the surrounding of 3000 agents.
If such a phenomenon enables us to answer to a part of the problems while making simulation less dependent to the numbers of elements, it raises the question of the accuracy of the physical approximations on agents having evolved so much.

1.4 Conclusion

In this paper a new hierarchical multi-agent model aiming to solve the inherent problems of the point-mass particle approach used by the cosmologists have been introduced. Qualitative evidence have been presented that a hierarchical multi-agent model is less sensitive to the number of elements used for a simulation than traditional models : beyond some threshold, emergent dynamics remain the same.
Future works include a quantitative analysis of the results presented here as well as a study of the complexity classes described by a wide parametrization of this model : by replacing the universe *as-we-know-it* in a larger picture of universe *as-it-could-be*, we will try to understand patterns formation and dynamical evolution. As it exists a difference between observation and numerical simulation on spheroidal gallactic formation (radial velocity of peripheral elements does not match with the observable one [1]) we will check if the model defined in this paper can bring some answers.

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