Evolving dissipative systems with discrete appearance: anisotropy and spatial scaling

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1. Introduction

Certain dynamic systems seem to evolve in discrete steps: these steps often consist of clearly distinguishable energy dissipation events of different sizes, occurring at variable time intervals. Earthquakes are typical examples of such events: the time scales involved in accumulation processes are different from those dominating dissipation events. Similarly, energy that gradually accumulates in plants is released during events consisting of forest fires. One may thus speak about dissipative systems with “discrete appearance”.

Such short-lived events not only fulfill a dissipation role in this context: they also change the system in which they occur, in ways that are consequential for future dissipation events. Fault structure and stress distribution are changed by earthquakes, and the quality of biomass and its spatial distribution are modified by forest fires. Irreversible changes in such systems influence their subsequent dynamics, which in its turn leads to other changes, produced by other events. Expectedly, these processes eventually lead to patterns that may be recognized as part of a system “fingerprint” based on spatial, temporal, and energy related aspects. This kind of task has been approached with different conceptual and methodological tools. Concentrating on temporal aspects, Carbone et al. (2005) identified a scaling law for waiting times between seismic events. Tiwari and Lakshmi (2005) constructed a “dynamical pattern” for different tectonic zones, based on earthquake monthly frequency data. Applying a space-time combined correlation integral to global seismicity data, Tosi et al. (2004) discovered a relation between the spatial influence length of an event and the time elapsed from the event. A different way to approach the multi-parametrical nature of a dynamic fingerprint is to apply Events Thread Analysis (ETA), which considers earthquake parameters such as space, time, and event size together, and can be used in comparisons between different seismically active zones (Suteanu et al., 2001). This approach was subsequently expanded, to address the relationship between scaling and anisotropy (Suteanu and Ioana, 2006, submitted).

This paper presents a methodological framework for the identification of the changing dynamic fingerprint associated with an evolving system. Relying on the principles of ETA, the proposed approach makes use of information regarding spatial location, spatial orientation, and temporal sequence involved in series of events. The methodology is applied to seismicity data related to volcanism in Hawaii.

2. The dynamic fingerprint of evolving dissipative systems

The methodology presented in this paper relies on two categories of operations: the construction of events threads, and their multiscale analysis, which are described below. They are applied here together with an algorithm designed to identify the orientation dependence of the dynamic fingerprint and its relation to scaling aspects, after which the fingerprint changes are monitored in time.

Designed to address irreversible processes and their discrete series of events, ETA starts from the premise that the succession of events in time is important. The method relies on the connection of points – representing events – in their order of occurrence. These points may be represented in multidimensional phase space (Suteanu et al., 2001): the resulting thread is then analyzed. For the purpose of this study, which focuses on spatial aspects, three-dimensional physical space will be used in ETA, as explained below.

Given the initial time series a_i corresponding to an events thread, it is first normalized, by subtracting the mean from all elements and by dividing each element by the standard deviation of the time series. This makes the different events threads comparable regardless of their initial value range. Based on the resulted vector v_i, the “trajectory” or “landscape” of the time series is built:

$$Q(i) = \sum_{k=1}^{i} v_k \qquad (1)$$

Figure 1 presents an events thread landscape example for earthquakes in Hawaii. To illustrate the idea underlying the method, the thread was composed from the landscape components *Q(i)* corresponding to the three spatial coordinates of the hypocentres. The

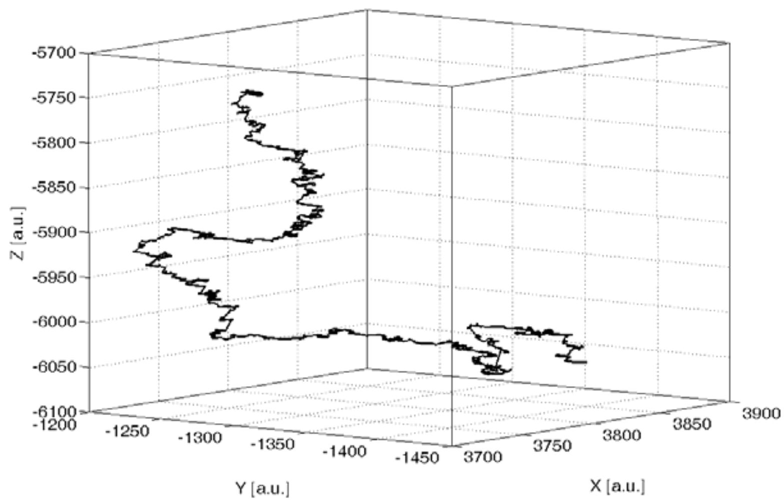


Figure 1. Events thread segment corresponding to earthquake hypocenter locations in Hawaii, considered in their temporal succession (arbitrary units).

time series obtained from events thread projections are then subject to multiscale analysis. To distinguish long-range correlations in the signal from trends of different categories, detrended fluctuation analysis (DFA) was the main method applied in this study. This method was introduced by Peng et al. (1994) with the aim of filtering out trends and of characterizing long-range correlations in time series affected by non-stationarities. The method was successfully applied to a wide range of situations (Bunde and Havlin, 2003; Movahed et al., 2006), and improved from the point of view of its reliability and accuracy (Kantelhardt et al., 2001; Kiraly and Janosi, 2005). The procedure applied in this study consists of the following steps:

The time series $Q(i)$ is divided into N non-overlapping sections of length s. For each section m (m = 1.. N), the trend of the data is determined using a least-square fit based on the polynomial $p_m(i)$. The detrended section is given by the difference between the original segment and the interpolated section:

$$Q_s(i) = Q(i) - p_m(i) \qquad (2)$$

The interpolation can be linear, quadratic, cubic, or higher order: the degree n of the polynomial may be changed, to identify the existence of different types of trends, which leads to methods denoted by DFAn: in the present study, DFA1 to DFA7 were used. DFAn removes trends of order n in the landscape time series $Q(i)$ and of order n-1 in the initial time series a_i. The variance of the resulting detrended time series $Q_s(i)$ is then calculated for each section of length s:

$$F_s^2(m) = \langle Q_s^2(i) \rangle = \frac{1}{s}\sum_{i=1}^{s} Q_s^2\left[(m-1)s + i\right] \qquad (3)$$

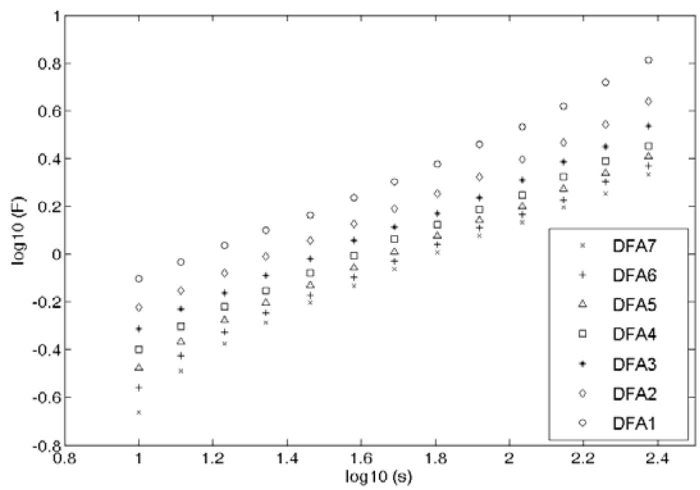


Figure 2. Detrended fluctuation analysis used in ETA for the seismicity in Hawaii.

Then the square root of the average of $F_s^2(m)$ over all sections is calculated:

$$F(s) = \left[\frac{1}{r} \sum_{m=1}^{r} F_s^2(m) \right]^{\frac{1}{2}} \tag{4}$$

where r represents the number of sections of size s. Actually, r can be larger than N/s: a larger number of sections reduces the noise level affecting the evaluation of the $F(s)$ relationship – for instance, the time series can be divided into sections of length s in different ways, beginning from different starting points in the time series (Kiraly and Janosi, 2005). This technique was used in the present study. The steps shown in equations (2)..(4) are repeated for different polynomial degrees n, which leads to a series of fluctuation functions $F^{(n)}(s)$. Long-range correlations in the time series can be identified if a power law dependence of the fluctuation function $F(s)$ on the section length s is found:

$$F^{(n)}(s) \propto s^K \tag{5}$$

For data lacking long-range correlations, one would have $K = 0.5$. If $K > 0.5$, the time series is characterized by persistence, which means that increasing (decreasing) values have the tendency to keep increasing (decreasing, respectively). $K < 0.5$ would indicate antipersistence: changes in one direction (e.g. increasing) have the tendency to be followed by changes in the opposite direction (e.g. decreasing) (Kantelhardt et al., 2001). By performing the described operations for different polynomial degrees n, one can see if and how trends elimination depends on n. If one represents F as a function of s in log-log graphs, the slope of the linear fit gives the value of K. Figure 2 shows an application example referring to an events thread segment from Hawaii. The artificial increase of the slope for polynomials of a low degree vanishes for higher values of n.

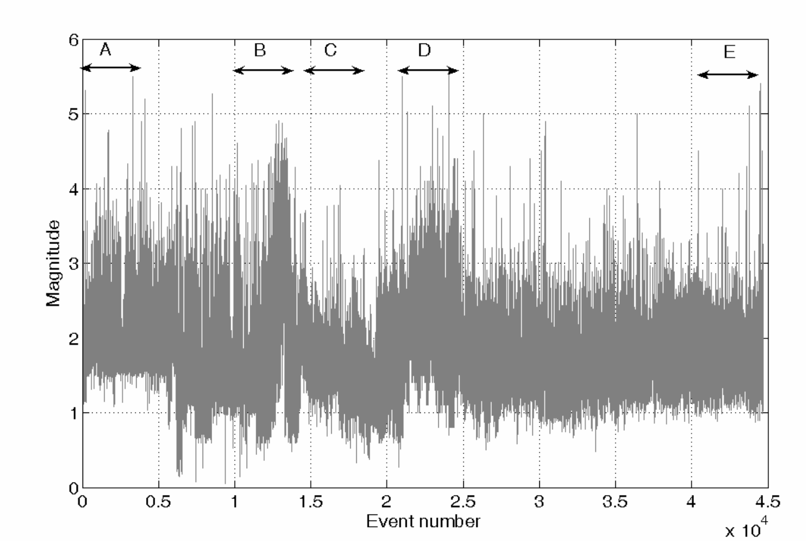


Figure 3. Magnitude of successive events in Hawaii. The five selected intervals are indicated.

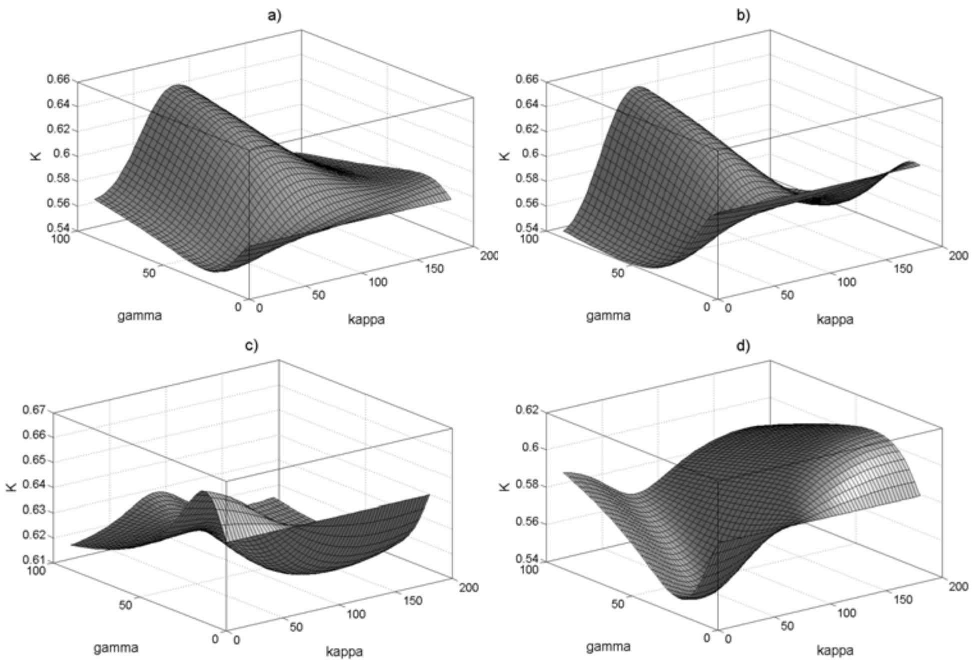
Deviations from a straight line occurring for small or too large ($s > N/20$) values of s are expected (Kantelhardt et al., 2001) and only the power law dependence interval is considered for the analysis. In the present study, the correlation for the determination of K was always very strong (correlation coefficient higher than 0.999).

In order to evaluate the orientation dependence of scaling aspects, we project the events thread on different spatial directions. The chosen system of coordinates consists of the X-axis running west to east, the Y-axis running south to north, and the Z-axis pointing towards the centre of the planet. The direction of the line D on which the events thread is projected is defined by two angles: gamma, the angle made by line D with the Z-axis, and kappa, the angle between the X-axis and the projection of D in the XOY plane. The angles kappa and gamma are increased stepwise, with a step of 3 degrees. DFA is applied to each projection, and the resulting K value is then represented for each point in kappa-gamma coordinates (Fig. 4).

To monitor changes in the evolving system, the method is applied to different windows of the events thread. The resulting persistence-orientation diagrams can then be compared for different stages in the evolution of the system.

3. Application example: volcanism and seismicity in Hawaii

We have applied the described framework to the study of the dynamic fingerprint of seismicity in Hawaii. Earthquakes have here various possible causes, such as advancing magma, changes in the crustal stress distribution due to volcanic loading or thermal cooling (Karpin et al., 1987; Wolfe et al., 2004), flank stability variation produced by dike intrusion (Delaney and Denlinger, 1999), the changing interplay between stress produced by magma reservoir and regional stress fields (Karpin et al., 1987), etc.



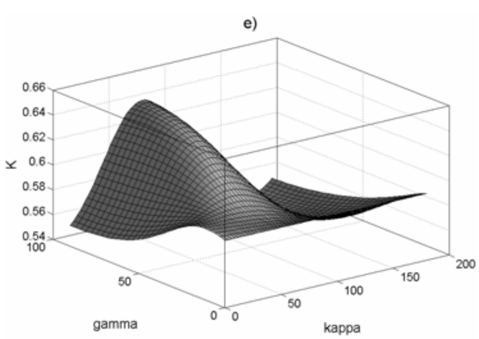


Figure 4. Stages of the evolving dynamic fingerprint of Hawaii. Events thread analysis applied to the five intervals (A to E) specified in Fig. 3. The fluctuation exponent K is shown for the spatial orientations defined by the angles kappa and gamma.

Ultimately, however, the underlying driving factor is represented by hot spot volcanism-related magmatic processes. Kilauea has been the most active volcano on the island in the last decades. Its Pu’u’O’o’-Kupaianaha eruption has been ongoing since January 1983. Volcano dynamics has significantly changed over time, shifting magma transport paths and vent locations, altering the configuration and the mass of lava ponds, etc. (Heliker and Brantley, 2004; for information regarding current processes, see HVO, 2006). Therefore, it was of particular interest to check whether the proposed methodology would reflect such changes based on dynamic fingerprints.

Seismicity data were acquired from the Advanced National Seismic System (ANSS) catalogue, for the area 18.5° to 20.5° N and 154.5° to 156.5° W, and the time interval 1989-2005. Five different segments of equal length (4096 events) were selected for this analysis (Fig. 3): as events threads, they cover different time intervals, depending on their temporal events density. Segments A (12.1989-12.1991) and E (10.2004-7.2005) represent the oldest and the most recent in the series, respectively; segments B (3.1995-12.1996) and D (6.1997-10.1999) include events of higher magnitude, whereas segment C (2.1997) corresponds to a particular eruption intensification, the so-called “episode 54” (HVO, 2006): all the events in this C segment occurred during only two months.

Applied to these data sets, the procedure presented above led to the results shown in Fig.4. In every case, the anisotropic character of spatial correlations is visible. Certain orientations emphasize persistence, with $K_{max} > 0.6$; for other orientations, K approaches values of uncorrelated noise. By comparing the dynamic fingerprints for different events thread segments, one can identify changes in orientation-related persistence. For example, segment B has an increased persistence in the vertical direction (gamma = 0), compared to segment A, without changing much its behaviour in near-horizontal direction (gamma = 90). On the contrary, segment C is characterized by a major pattern change; its overall persistence is higher, and its behaviour related to the whole spectrum of gamma values is different from those of the preceding segments; such changes can be studied by following cross-sections with constant kappa through persistence diagrams. During this intense eruption episode, persistence is particularly high for vertical and near-vertical orientations. Interestingly, after a new pattern change (segment D) corresponding to another earthquake swarm, the dynamic fingerprint is returning to a state that is similar to the initial ones (segment E). The three-dimensional surfaces representing dynamic fingerprints can be viewed from different angles: rotating them in kappa-gamma-K space provides a comprehensive comparative perspective upon the fingerprints of different time intervals; to reach a higher precision in these comparative studies, one may use contour plots to complement the three-dimensional representations.

The established dynamic fingerprints emphasize changes in system dynamics – they can be used in detailed studies regarding the correspondence between persistence, spatial orientation, and observed phenomena. Based on spatial data, they provide qualitatively new information about the evolving geosystem.

4. Conclusions

Applied together with a reliable multiscale evaluation method such as DFA, the proposed Events Thread Analysis methodology establishes a “dynamic fingerprint” of a seismically active zone and is able to discern its changes in time. Depending on the purpose of the investigation, dynamic fingerprints can be produced with the chosen resolution, which is defined by the angle step used in the ETA anisotropy determination procedure.

The methodology can be easily implemented, and it provides information that is difficult or impossible to discern from conventional multiscale analysis of the data. Moreover, since it relies on the identification of correlations in available data, its application does not require supplementary resources and/or data collection effort.

Starting from spatial information, it leads to insights regarding dynamic aspects of evolving systems. It can thus be used together with different types of approach, such as high-resolution tomography of volcanic structures, or spectral analyses of seismic events (Chouet, 2003). Applied in conjunction with geophysical and geological studies, this ETA-based methodology is expected to contribute to a well focused, fruitful study of such complex systems. Applications can focus on real natural systems, such as seismically active zones, areas affected by forest fires, etc., as well as on models: the identification of dynamic fingerprints for model outputs can support a useful, multi-perspective comparison with the modeled systems.

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