Computational Simulation of the Hydraulic Fracturing Process

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1. Introduction

The hydraulic fracturing process has been used since the first half of the past century for reservoir stimulation treatments. Its bases are simple: some fluid (usually water or mud) is injected into the reservoir at a given rate. At the well's bottom, the pressure begins to increase until it surpasses the elastic resistance of the surrounding rock, and a fracture begins to propagate through the formation. The fracturing fluid enters the fracture volume and suddenly the pressure drops. The direction for the fracture propagation is mainly perpendicular to the minimum principal stress (the minor of the two principal tectonical stresses) since it's the easiest path to follow [Johnson 1988]. Eventually the system enters in an equilibrium state between the fracturing pressure and the sum of internal stresses and pressures. At this point the fracture stops its evolution. Hopefully the fracture will connect zones of highly petroleum concentration with the extraction point. This is the principal use for these treatments [Economides 2000].

Of course, there are many variables in the process. For example we must consider the in situ stress state mentioned above. Another variable should be rock elastic behavior. But the most important variable is the pressure of the fracturing fluid which gives information about the evolution of the process. After the fracture starts to propagate the pressure begins to decrease as mentioned above. When the pressure reaches an equilibrium value, the process is terminated. Therefore, any model for the treatment shall include the pressure variable from the beginning. In this work, we show a model for the computational simulation of these and other variables relevant to the process in a bidimensional propagation scenario.

2. The model

In order to successfully reproduce the physics involving the hydraulic fracturing process, we must create a model for the geomechanics of the reservoir and a model for the dynamics of the fracturing pressure. We will begin illustrating the model that we develop for the rock.

2.1 The rock

Our model is based in a discrete element method developed for the study of fragmentation processes in the ICA (*Institute of computational physics*) [Kun 1996]. In this model the rock is divided in an array of irregular polygons of random configuration. We choose the Voronoi polygon method for our particular case. In this method a set of points, called *Voronoi Points*, are randomly distributed over the space of interest. The polygon associated to the Voronoi point **p** is the set of coordinates that fulfils the following condition:

$$V(\mathbf{p}) = \{\mathbf{r} \mid dist(\mathbf{r},\mathbf{p}) < dist(\mathbf{r},\mathbf{q})\} \qquad (1)$$

Where **q** is just another Voronoi Point. So a Voronoi polygon can be constructed like a Brilloin zone of the Voronoi points in 2 dimensions.

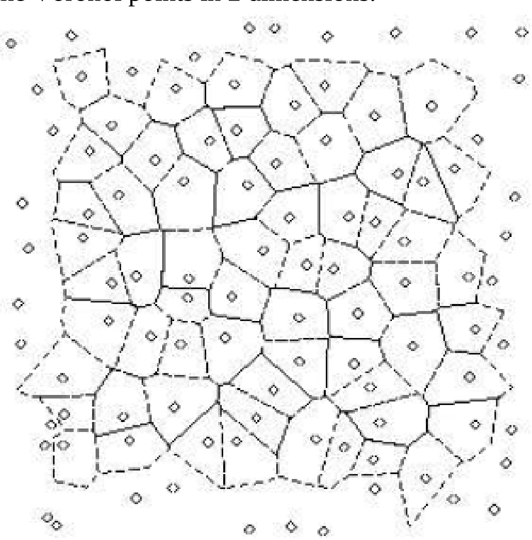

Figure. 1. An array of Voronoi polygons representing the rock. The Voronoi Points are also shown as circles.

The polygons suffer elastic forces with each other modeled by an elastic beam connecting both. Two adjacent polygons (polygons i and j) suffer a force proportional to the rock elastic constants given by a discretization of the Cosserat equations [Kun 1996]:

$$F_x^{\,i} = \frac{1}{a}(x_j - x_i)$$
$$F_y^{\,i} = \frac{1}{b + c/12}(y_j - y_i) + \frac{1}{b + c/12}\frac{l}{2}(\Theta_i + \Theta_j) \qquad (2)$$
$$M^{\,i} = \frac{1}{b + \frac{c}{12}}\frac{l}{2}(y_j - y_i + l\Theta_j) + \frac{1}{b + \frac{c}{12}}\left(\frac{b}{c} + 1/3\right)l(\Theta_j - \Theta_i)$$

Whit $a=l/EA$, $b=l/GA$ and $c=l^3/EI$ with A the length of the side that both polygons share, E is the Young modulus, G the shear modulus, I is the beam's moment of inertia for flexion, the coordinates (x_i,y_i,Θ_i) are the position of the center of mass and the angular displacement for the i-th polygon and l is the distance of equilibrium between the two center of masses. The coordinate system is such that the x axis is perpendicular to the side shared by the polygons. We have then two components for the force F_x and F_y and one flexural torque M around the center of mass axis. These cohesive forces should disappear once the beam between the polygons breaks. According to the Mohr Coulomb criterion [Economides 2000], the fracture starts when the strain surpasses certain threshold. In our model, the two adjacent polygons will feel no further cohesive force once one of the following three conditions is true:

$$\varepsilon_t > u_t, \varepsilon_s > u_s, \max(\Theta_j, \Theta_i) > u_\Theta \qquad (3)$$

Where ε_t and ε_s are the tensile and shear strain of the cohesive beam and the u values are parameters of the simulation.

Another force is the granular repulsion between polygons that opposes a force trying to join them. This force is proportional to the bulk modulus Y and the overlapping area of the two polygons S [Kun 1996]:

$$\mathbf{F}_g = \frac{YS}{L_c}\mathbf{n} \qquad (4)$$

The unit vector **n** is perpendicular to the line shown in Fig 2 and the length $L_c=0.5(1/r_i+1/r_j)$ whit r_i is the radius of the circle with the same area of the i-th polygon.

We also want to study the effect of tectonical stresses in the geometry of the fracture. These stresses are introduced by four rectangular polygons near each of the four boundaries. The rectangles and polygons suffer the same granular repulsion and the tectonical stresses are applied to the rectangles. As the rectangles move in time, they strain the whole rock as the tectonical stresses do in real life.

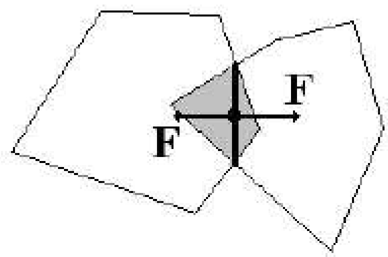

Figure. 2. Granular Force for overlapping polygons. The force is applied in the middle point of the line joining the intersection points and its direction is perpendicular to this line with repulsive sign. The force is also proportional to the overlapping area (grey).

All of these forces are conservatives ones. We need a proper viscosity force to ensure that the system reaches equilibrium at a given time. Based on a viscoelastic model, we choose the simplest force of this kind:

$$\mathbf{F}_{\boldsymbol{\mu}} = -C\mathbf{v} \qquad (5)$$

The constant C is a parameter of our simulation with a set value that ensures a reasonable time for the system to enter its equilibrium state (In real petroleum wells a process lasts 30 minutes to 2 hours) [Economides 2000].

2.2 The fluid

The only effect, in our model, that the fluid has on the polygons is just another force given by its pressure. This force is given by:

$$\mathbf{F}_p = p(t)\,l\,\mathbf{n} \qquad (6)$$

Where p(t) is the fluid pressure in time, *l* is the area of the surface in contact with the fluid (the length of the side in contact with the fluid in our two dimensional case) and **n** is an unit vector perpendicular to the contact side.

We assume that the functional dependence of the fluid pressure with time varies as the pressure of a hydraulic column with height h_0 at the initial time. Considering that the pressure decreases when the fracture begins to propagate, we propose the following relation.

$$p(t) = \rho g h_0 - \rho g \frac{V(t)}{S} = p_0 - \rho g \frac{V(t)}{S} \qquad (7)$$

Whit ρ is the density of the fracturing fluid, g the gravity acceleration, V(t) the volume of the opened fracture at time t and S the cross section of the injection pipe

2.3 Molecular dynamics

We have now all the forces that each polygon suffer at a given time. With the net force calculated, we may solve Newton's second law equation and find the position (both, angular displacement and the center of mass coordinates) in the next time step of our simulation. We repeat this procedure for each polygon and obtain the final configuration of the whole system. The method is called Molecular Dynamics and requires that we calculate the position at time t+Δt with the following formula:

$$\mathbf{r}(t+\Delta t) = 2\mathbf{r}(t) - \mathbf{r}(t-\Delta t) + \frac{\mathbf{F}(t)}{m}\Delta t^2 + O(\Delta t^4) \qquad (8)$$

This is the Verlet formula for M.D [Verlet 1967]. A similar equation is found for the angular dynamics of the polygons.

3. Results

We did a simulation for an stress-free array of 20x20 polygons until the pressure of the fracturing fluid reached the equilibrium valor. The fracture obtained is shown on Fig 3. We can se that without any external stresses, the direction of fracture propagation is almost random.

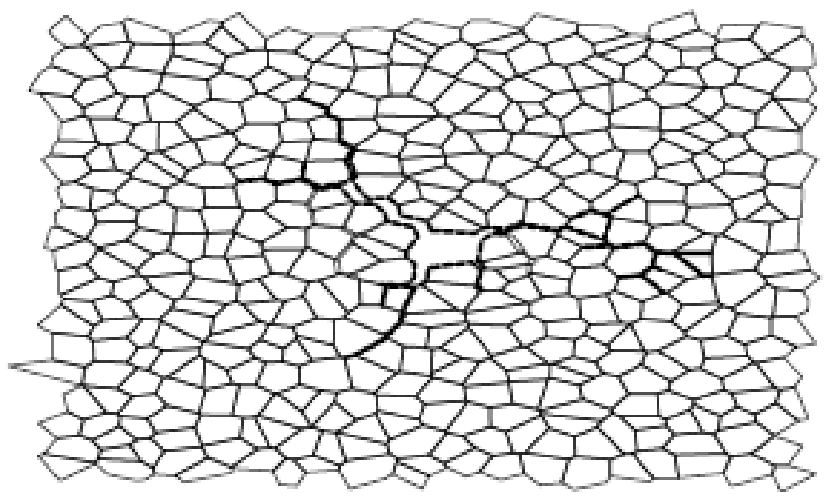

Figure. 3. Equilibrium fracture obtained for a 20x20 polygon array.

By means of the box-counting algorithm we can calculate the fractal dimension for the perimeter [Bunde 1996]. We obtain a fractal dimension of 1.223(4). Of course this fractal dimension should depend on the resolution of the grid. So, we ran several calculations for different resolutions (15x15 polygons, 20x20 and so on) and see that the fractal dimension has an asymptotic limit with a given exponential fit (see Fig 4).

This limit has a value of 1.311 which is in close agreement with the value 1.39 of previous simulations using a similar model [Tzschichholz 1995]. So, in order to successfully reproduce the fractal geometry we don't need resolutions of higher order than 35x35 polygons by the extrapolation of our results.

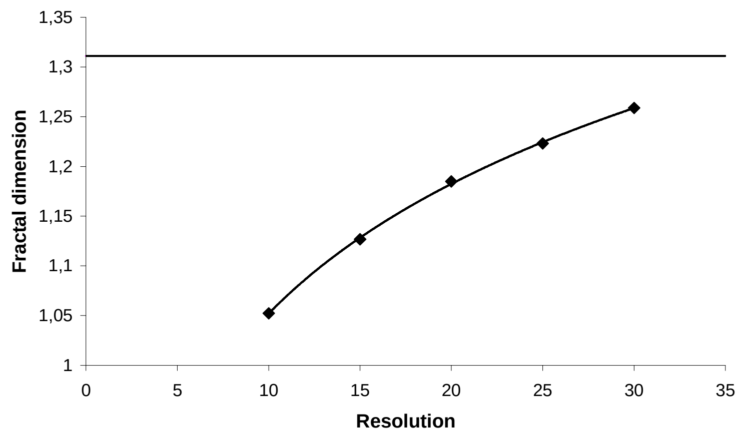

Figure. 4. Fractal dimension dependence with grid resolution

The pressure of the fluid versus time is shown in Fig 5 where we can see that eventually it reaches an equilibrium value. With limestone elastic constants, we find that the pressure reaches a constant value at 1200 seconds or 20 minutes.

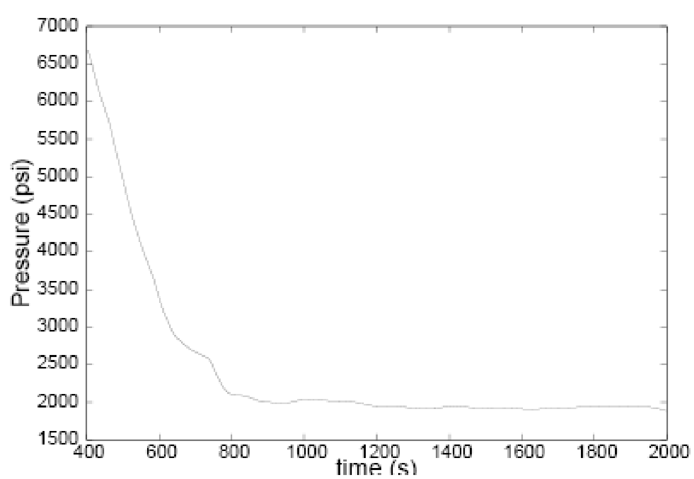

Figure. 5 Evolution of the fracturing fluid pressure with time.

Now we may introduce the tectonical stresses to find the dependence on the fracture geometry. We start with a simulation of a 25x25 array of polygons with and horizontal stress of 5000psi and a vertical stress of the same value (Fig 6 above), then we set an horizontal stress of 10000 psi (Fig 6 center) and finally of 15000 psi (Fig 6 below). We see that the higher the difference between the principal stresses, the more defined is the path for the fracture propagation along the horizontal axis.

Figure 6. Three different fractures for tectonical stresses of 5000 psi along the y axis and 5000 (above), 10000 (center) and 15000 psi (below) for the stress along the x axis which is always the maximum stress.

4. Conclusions

We have presented a model for the computational simulation of the hydraulic fracturing process using a discrete element method that reproduces the elastic behavior and the fracture criterion of the rock. The fracturing fluid pressure is modeled as the one of an hydraulic column. Its height decreases as the fracture volume increases hence the pressure drops just as happen in the real fracturing process.

We also show that the fracture geometry depends of the in situ stress state as expected by geomechanical theory. This is an example of the less action principle since the path the propagation follows only opposes the minimum tectonical stress.

Future simulation on the subject should include a better and more refined model for the fluid considering factors like the viscocity of the fluid and the dynamics inside reservoir porous media.

5. References

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