

# The Role of Information 'Barriers' in Complex Dynamical Systems Behavior

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In this supposed 'information age' a high premium is put on the widespread availability of information. Access to as much information as possible is often cited as key to the making of effective decisions. Whilst it would be foolish to deny the central role that information and its flow has in effective decision making, this paper explores the equally important role of 'barriers' to information flows in the robustness of complex systems. The analysis demonstrates that (for simple Boolean networks at least) a complex system's ability to filter out, i.e., block, certain information flows is essential if it is not to be beholden to every external signal. The reduction of information is as important as the availability of information.

## 1 Introduction

In the Information Age the importance of having unfettered access to information is regarded as essential - almost a 'right' - in an open society. It is perhaps obvious that acting with the benefit of (appropriate) information to hand results in 'better' actions (i.e., actions that are more likely to achieve desired ends), than acting without information, although incidents of 'information overload' and 'paralysis by (over) analysis' are also common. In this chapter, I would like to examine one particular aspect of information: how barriers to information, and its flow, are essential in the

maintenance of a coherent functioning network. The paper begins with an introduction to Boolean networks and particular properties that are relevant to our analysis herein.

## 2 Boolean networks: Their structure and their dynamics

Given the vast number of papers already written on both the topology and dynamics of such Boolean networks there is no need to go into too much detail here. The interested reader is encouraged to look at Kauffman [1] for his application of Boolean networks to the problem of modeling genetic regulatory networks. A short online tutorial is offered by Lucas [2].

The state, or configuration, space for such a network contains  $2^N$  unique states, where  $N$  is the size of the. Because state space is finite, as the system is stepped forward it will eventually revisit a state it has already visited. Combine this with the fact that from any state the *next* state is unique, then any Boolean network will eventually follow a cycle in state space. As a result, state space (or phase space) is connected in a non-trivial way, often containing multiple periodic attractors each surrounded by fibrous branches of states, known as *transients*. The transition functions for each node were chosen randomly for the networks examined in this research, but they did *not* include the constant rules 0 (0000), or 15 (1111) as these force the node to be input-independent - nodes with constant transition functions do not change from their initial state.

How is a network's structure and its dynamics related? It is in the consideration of this question that we can begin to explore the role of information flows in such systems.

The potentially complex dynamics that arises from the apparently simple underlying structure is the result of a number of *interacting* structural feedback loops. It is the flow of information around these structural loops, and the interactions between these loops that results the complex structure of state space. The number of structural  $P$ -loops increases exponentially (on average) as  $N$  increases, for example, whereas the number of state space  $p$ -loops increases (on average) in proportion to  $N$  (not  $\sqrt{N}$  as reported in [1], which was found to be the result of sampling bias [3].

For networks containing only a few feedback loops it is possible to develop an algebra that can relate  $P$ -space to  $p$ -space. However, as the number of  $p$ -loops increases this particular problem becomes intractable very quickly indeed, and the development of a linking algebra utterly impractical.

Sometimes the interaction of a network's  $P$ -loops will result in  $p$ -space collapsing to a single period-1 attractor in which every point in state space eventually reaches the same state. Sometimes, a single  $p$ -loop will result whose period is much larger than the size of the network - such attractors are called *quasi-chaotic* (which can be used as very effective random number generators). Most often multiple attractors of varying periods result which are distributed across state space in complex ways.

Before moving on to consider the robustness of Boolean networks, which will then allow us to consider the role information barriers play in network dynamics, it should be noted that state space can also be considered to be *functional space*. The different attractors that emerge from a network's dynamics represent the network's

different functions. For example, in Kauffman's analogy with genetic regulatory networks, the network structure represents the *genotype*, whereas the different phase space attractors represent the resulting *phenotype*, with each attractor representing a different cell type. We can also regard the different attractors as different modes of operation. Furthermore, an appreciation of a system's state space structure tells us about the different responses a system will have to a variety of external perturbations, i.e., it tells us which *contextual archetypes* the system 'sees' and is able to respond to.

## 2.1 Defining dynamical robustness

The dynamical robustness of networks is concerned with how stable a particular network configuration is under the influence of small external perturbations. In Boolean networks we can assess this measure by disturbing an initial configuration (by flipping a single bit/input, i.e., reversing the state of one of the system's nodes) and observing which attractor basin the network then falls into. If it is the same attractor that follows from the unperturbed (system) state then the state is stable when perturbed in this way. An average for a particular state (or, system configuration) is obtained by perturbing each bit in the system state and dividing the number of times the same attractor is followed by the network size. For a totally unstable state the robustness score would be 0, and for a totally stable state the robustness score would be 1. The dynamical robustness of the entire network is simply the average robustness of every system state in phase space. This measure provides additional information concerning how state space is connected in addition to knowing the number of cyclic attractors, their periods, and their weights (i.e., the volume of phase space they occupy).

## 2.2 More on structure and dynamics: Walls of constancy, dynamics cores and modularization

As we have come to expect in complex systems research, there is always more to the story than what first meets the eye. This is also the case with the issue of the relationship between network structure and dynamics. Although information flows (and is transformed) around the various structural networks, certain logical couplings 'emerge' that force particular nodes into one state or another, keeping them in that state for as long as the network is simulated. In other words, once the network is initiated and run forward, after some seemingly arbitrary period some nodes cease to allow information to pass. These 'fixed', or 'frozen' nodes effectively disengage all structural feedback loops that include those particular nodes - although these structural loops still exist, they are no longer able to carry information around them. As such we can refer to them as *non-conserving information* loops. A number of such nodes can form 'walls of constancy' through which no information can pass, and effectively divide the network up into non-interacting sub-networks, i.e., the network is *modularized*. This process is illustrated in Figure 1.

The identification of these nodes is non-trivial (and even 'non-possible') before the networks are simulated. Although the effect is the same as associating a constant transition function with a particular node, the effect 'emerges' from the dynamic interaction of the structural feedback loops. It is a rare case indeed that these

interactions can be untangled and the emerging frozen nodes identified analytically beforehand.

These ‘frozen’ nodes do not contribute to the qualitative structure of phase space, or network function. A Boolean network characterized by a  $2p_1, 1p_2$  phase space, say, will still be characterized by a  $2p_1, 1p_2$  phase space after the frozen nodes are removed. In this sense, they don’t appear to contribute the networks function - they block the flow of information, but from this perspective have no functional role. Another way of saying this is that, if we are only concerned with maintaining the qualitative structure of phase space, i.e., the gross functional characteristics of a particular network, then we need only concern ourselves with *information conserving* loops; those structural feedback loops that allow information to flow around the network.

The process employed to identify and remove the non-conserving loops is detailed in [4]. As network size increases it becomes increasingly difficult to determine a network’s phase space structure. As such reduction techniques are not only essential in facilitating an accurate determination, but also in research that attempts to develop a thorough understanding of the relationship between network structure and network dynamics (as, already mentioned, only information conserving structural loops contribute to the network’s gross functional characteristics). Another term: what remains after all non-conserving loops (which includes associated ‘frozen’ nodes, and nodes that have no outgoing connections, or ‘leaf’ nodes) are removed from a particular network is known as the network’s *dynamic core*. A dynamic core contains only information conserving loops. So the majority of (random) Boolean networks comprise a dynamic core (which may be modularized) plus additional nodes and connections that do not contribute to the asymptotic dynamics of the network.

### **3 The role of non-conserving (structural) information loops**

As the gross phase space characteristics of a Boolean network and its ‘reduced’ form are the same, i.e., (in this sense) they are *functionally equivalent*, it is tempting to conclude that non-conserving information loops - information barriers - are irrelevant. If this were the case then it might be used to support the widespread removal of such ‘dead wood’ from complex information systems, e.g., human organizations. The history of science is littered with examples of theories which once regarded such and such a phenomena as irrelevant, or ‘waste’, only to discover later on that it plays a very important role indeed. The growing realization that ‘junk DNA’ is not actually junk is one such example. What is often found is that a change of perspective leads to a changed assessment. Such a shift leads to a different characterization of non-conserving information loops. Our limited concern, thus far herein, with maintaining a qualitatively equivalent phase space structure in the belief that a functionally equivalent network is created, supports the assessment of non-conserving information loops as ‘junk’. However, this assessment is wrong from a different perspective.

There are at least two roles that non-conserving information loops play in random

Boolean networks:

1. The process of modularization, and;
2. The maximization of robustness.

### 3.1 Modularization in Boolean networks

We have already briefly discussed the process of modularization above. This process, which we might label as an example of *horizontal emergence* [5], was first reported in [6]. It was argued that the spontaneous emergence of dynamically disconnected modules is key to understanding the complex (as opposed to ordered and quasi-chaotic) behavior of complex networks. So, the role of non-conserving information loops is to limit the network's dynamics so that it does not become overly complex, and eventually quasi-chaotic (which is essentially random in this scenario: when you have a network with a high-period attractor of say  $10^{20}$  - which is not hard to obtain - it scores very well indeed against all tests for randomness. One such example is the *lagged Fibonacci* random number generator).

In Boolean networks the resulting modules are *independent* of each other, so the result of modularization, is a collection of completely separate subsystems. This absolute independency is different from what we see in nature, but the attempt to understand natural complex systems as integrations of *partially independent* and interacting modules is arguably a dominant theme in the life science, cognitive science, and computer science (see, for example, [7]). It is likely that some form of non-conserving, or perhaps 'limiting', information loop structure plays an important role in real world modularization processes. Another way of expressing this is: *organization is the result of limiting information flow.*

### 3.2 Dynamic robustness of complex networks

Dynamic robustness was defined above, in a different way, as the stability of a network's qualitative behavior in the face of external signals. In this section we will consider the dynamic robustness of an ensemble of random Boolean networks and their associated 'reduced' form to assess any difference between the two.

To do this comparison the following experiment was performed. 110,000 random Boolean networks with  $N=15$  and  $k=2$  (with random connections and random transition functions, excluding the two constant functions) were constructed. For each network its reduced form was determined using the method detailed in [4]. The average dynamical robustness was calculated for both the unreduced networks and their associated reduced networks. The data from this experiment is presented in Figure 2a, which shows the relationship between the average unreduced and reduced dynamic robustness for the 110,000 networks considered (the average is represented by the black line). It is clear that on average the robustness of the reduced networks is noticeably lower than the unreduced networks. *On average*, the dynamic robustness of the reduced networks is typically of the order of 20% less than their parent (unreduced) networks. Of course, the difference for particular networks is dependent on specific contextual factors such as the number of non-conserving information loops in the (unreduced) networks - the extent of the dynamic core, in

other words. This strongly suggests that the reduced networks are rather more sensitive to external signals than the unreduced networks. In some instances the robustness of the reduced network is actually zero meaning that any external perturbation whatsoever will result in qualitative change. What is also interesting, however, is that sometimes the reduced network is actually more robust than the unreduced network. This is a little surprising, but not when we take into account the complex connectivity of phase space for these networks. This effect is observed in cases when there is significant change in the relative attractor basin weights as a result of the reduction process and/or a relative increase in the orderliness of phase space.

The different colors in Figure 2a indicate the number of data points that fall into a particular data bin (the data is rounded down to 2 decimal places, i.e., 100 data bins are created). So red regions contain many data points ( $> a^{10}$  where  $a$  is the *relative temperature* of the data,  $a = 1.8$  in this case) and dark blue regions contain only one (or  $a^0$ ) data point. It is clear that the data is multi-modal and as such we must be wary of using averages. Whereas Figure 2a includes all the data collected regarding unreduced and reduced dynamic robustness, Figures 2b – 2o show only the data for particular sizes of dynamic core. This helps considerably in understanding the detailed structure of Figure 2a. The various diagonal ‘modal peaks’ relate to networks with different dynamic core sizes, and the different horizontal structures correlate with networks containing smaller dynamic cores which can only display a fewer number of dynamic robustness values, i.e., as the size of the dynamic core decreases the data appears more discrete, and as the core size increases the data appears more continuous (although there is still an upper bound to its resolution).

Further analysis was performed to confirm the relationship between network structure, dynamic core structure and, phase space structure. This included comparing the number of structural feedback loops in the overall network to the number of (information conserving only) loops in network’s dynamic core. Figure 3 shows the data for all ( $N=15, k=2$ ) networks studied with all dynamic core sizes. The data indicates that on average the dynamic core of a network has between 30% and 60% fewer structural feedback loops; all of them information conserving loops. In the next section we shall consider the implications of this in terms of phase space characteristics and dynamic robustness.

## 4 Phase space compression and robustness

In Boolean networks, each additional node doubles the size of phase space. So even if ‘frozen’ nodes contribute nothing to longer term (asymptotic) dynamics, i.e., the number and period of phase space attractors, they at least increase the size of phase space. For example, the phase space of an  $N=20$  network is 1024 times larger than an  $N=10$  network. Thus, node removal significantly reduces the size of phase space. As such, the chances that a small external signal will inadvertently target a sensitive area of phase space, i.e., an area close to a separatrix, and therefore pushing the network into a different attractor, are significantly increased: a kind of *qualitative chaos*. This explains why we see the robustness tending to decrease when non-conserving information loops are removed.

Prigogine said that self-organization requires a container (self-contained-organization). The non-conserving information loops function as the environmental equivalent of a container. So it seems that, although non-conserving information loops do not contribute to the long term behavior of a particular network, these same loops play a central role as far as network stability is concerned. Any management team tempted to remove 80% of their organization in the hope of still achieving 80% of their yearly profits (which is sometimes how the 80:20 principle in general systems theory is interpreted) would find that they had created an organization that had no protection whatsoever to even the smallest perturbation from its environment – it would literally be impossible to have a stable business.

It should be noted that the non-conserving information loops do not act as impenetrable barriers to external signals (information). These loops simply limit the penetration of the signals into the system. For example, in the case of the modularization process, the products of incoming signals may, depending on network connectivity, still be fed from a *non-conserving* information loop into information *conserving* loops for a particular network module. Once the signals have penetrated a particular module, they cannot cross-over into other modules (as the only inter-modular connections are via non-conserving loops).

It should also be noted that, even though a particular signal may not cause the system to jump into a *different* attractor basin, it will still push the system into a different state on the *same* basin. The affect of signals that end-up on non-conserving information loops is certainly not nothing. So, although I use the term information ‘barriers’, these barriers are semi-permeable.

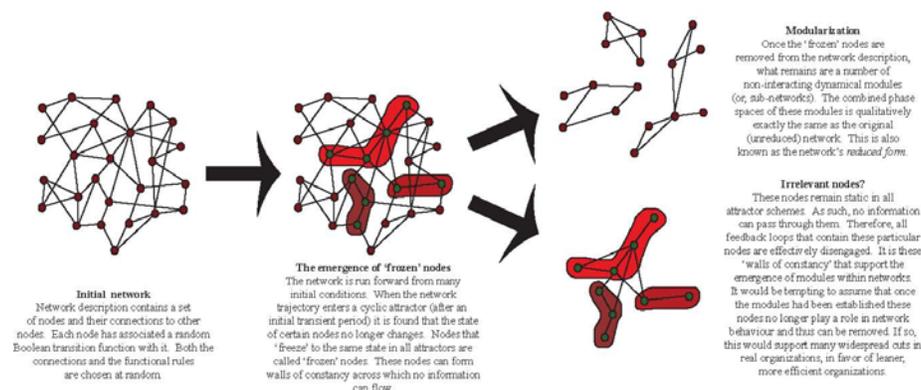
## 5 Conclusions

From the analysis presented herein it is clear that non-conserving information loops - information barriers - are not ‘expendable’: they protect a system from both *quantitative* and *qualitative* (quasi) chaos. Quantitative chaos is resisted by the emergent creation of modules - through the process of modularization - which directly reduces the chances of phase space being dominated by the very long period attractors associated with quasi-randomness in Boolean networks. Whereas qualitative chaos - the rapid ‘jumping’ from one attractor basin to another in response to small external signals - is resisted by the expansion of phase space, which reduces the possibility of external signals pushing a system across a phase space separatrix.

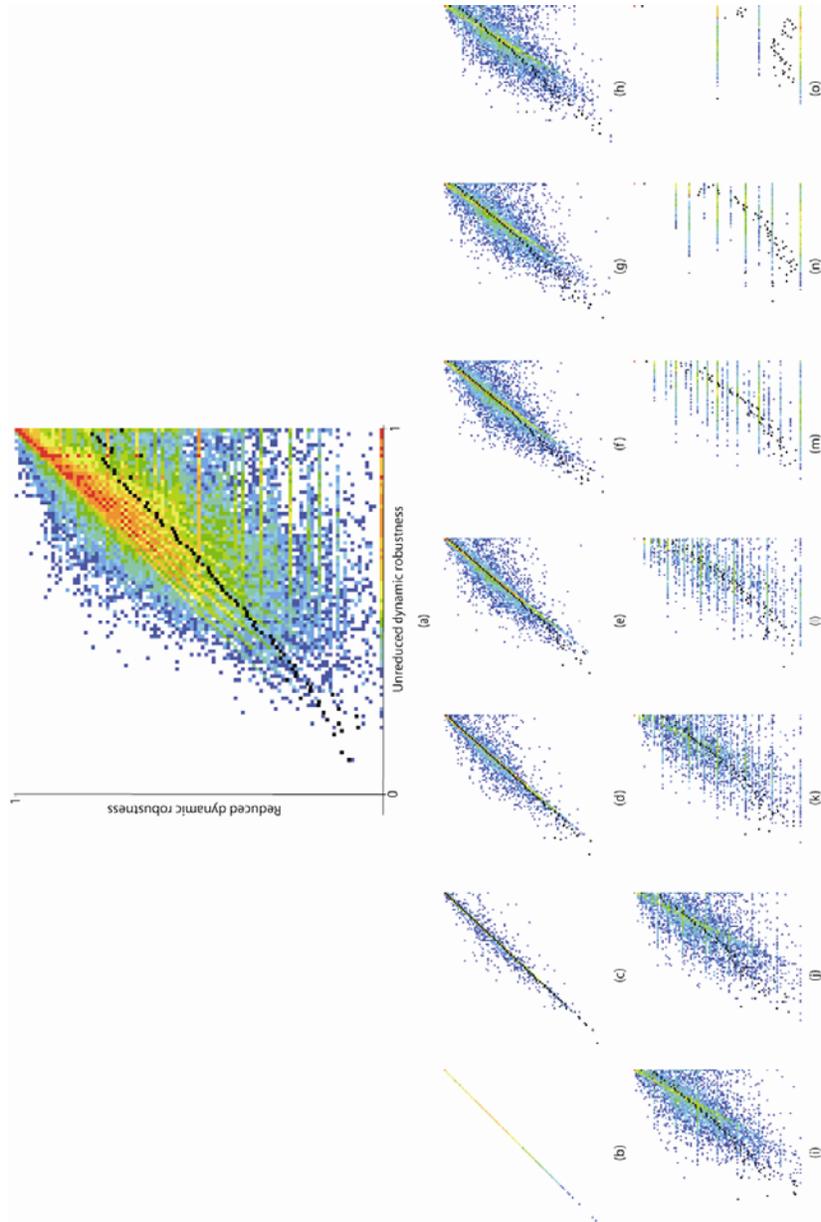
To conclude, I would suggest that ‘barriers’ (both impenetrable and semi-permeable) to information flow play an important role in the functioning of all complex information systems. However, the implications (and meaning) of this for real world systems is open to many different interpretations. At the very least it suggests that ‘barriers’ to information flow should be taken as seriously as ‘supports’ to information flow (although, paradoxically, a good ‘supporter’ is inherently a good ‘barrier’!)

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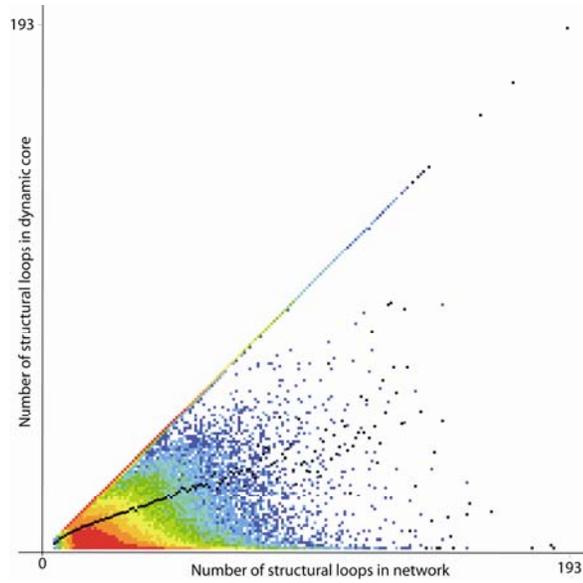
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**Figure 1** *The process of modularization in complex networks*



**Figure 2** A detailed view of the dynamical robustness data collected. (a) all data, with black points showing the overall average, (b) data associated with a dynamic core size of 15, (c) 14, (d) 13, (e) 12, (f) 11, (g) 10, (h) 9, (i) 8, (j) 7, (k) 6, (l) 5, (m) 4, (n) 3, and (o) 2.



**Figure 3** A data histogram showing the relationship between the number of structural feedback loops in the unreduced networks, and the number of active structural loops in their dynamic cores. The black points represent the average number of structural loops in dynamic core.