

Chapter 1

Exergy Sustainability for Complex Systems

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Exergy is the elixir of life. Exergy is that portion of energy available to do work. Elixir is defined as a substance held capable of prolonging life indefinitely, which implies sustainability of life. In terms of mathematics and engineering, exergy sustainability is defined as the continuous compensation of irreversible entropy production in an open system with an impedance and capacity-matched persistent exergy source. Irreversible and nonequilibrium thermodynamic concepts are combined with self-organizing systems theories as well as nonlinear control and stability analyses to explain this definition. In particular, this paper provides a missing link in the analysis of self-organizing systems: a tie between irreversible thermodynamics and Hamiltonian systems. As a result of this work, the concept of “on the edge of chaos” is formulated as a set of necessary and sufficient conditions for stability and performance of sustainable systems. In addition, exergy is shown to be a fundamental driver and necessary input for sustainable systems, since exergy input in the form of power is a single point of failure for self-organizing, adaptable systems.¹

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1 Introduction

Exergy is the elixir of life. Exergy is that portion of energy available to do work. Elixir is defined as a substance held capable of prolonging life indefinitely, which implies sustainability of life. In this paper simplified models are developed and utilized to address the notion of sustainability of a lifestyle. The purpose of this paper is to present the concept of exergy sustainability based on irreversible and non-equilibrium thermodynamics combined with some concepts from self-organization and adaptivity as well as nonlinear stability and control. The result of this synthesis is the mathematical definition of sustainability based on a fundamental driver and input, exergy rate, to a self-organizing system. In fact, exergy rate (power) input, is a single point failure. The flow of exergy into an open system that is continuously undergoing self-organization and adaptivity determines whether the system will persist or disintegrate. As a result, the balance of exergy flows into and out of the system versus the exergy consumption, irreversible entropy production, in the system will be studied.

Traditionally, exergy concepts are founded in the first and second laws of thermodynamics in the field of physics. However, these laws have both economical and environmental significance as well and can be applied in a more universal manner. Assessment of economic factors, that are based on the second law of thermodynamics are: i) exergy is not conserved and ii) exergy can be used as a common measure of resource quality along with quantity (i.e., materials and energy) [1]. Exergy from a physics standpoint is formally defined as the maximum amount of *work* that a subsystem can do on its surroundings as it approaches thermodynamic equilibrium reversibly or the degree of distinguishability of a subsystem from its surroundings[1]. Therefore, exergy can be used to measure and compare resource inputs and outputs which include wastes and losses [1]. For economic processes exergy is consumed not conserved. In addition, exergy can be used as a measure of assessing technical progress for economic growth theory. In this paper the basic concepts from [2] and [3] will provide the background to explain the definition of exergy sustainability: the continuous compensation of irreversible entropy production in an open system with an impedance and capacity-matched persistent exergy source.

This paper is divided into seven sections. Sections 2 and 3 provide the preliminary thermodynamics and Hamiltonian mechanics definitions. Section 4 develops the relationships and connections between thermodynamics and Hamiltonian mechanics. Section 5 defines the necessary and sufficient conditions for stability of nonlinear systems. Section 6 discusses self-organizing, adaptive systems and combines the concepts of the previous sections in order to explain the definition of exergy sustainability. Finally, Section 7 summarizes the results with concluding remarks.

2 Thermodynamic Concepts

In this section, results from the first and second laws of thermodynamics are used to define exergy. A corollary of the first law of thermodynamics states energy is conserved (see Fig. 1.1 - left). A corollary of the first and second laws of thermodynamics state that the entropy of the universe must be greater than or equal to zero. Conservation of energy can be written in terms of energy rate for a system [4]

$$\dot{\mathcal{E}} = \sum_i \dot{Q}_i + \sum_j \dot{W}_j + \sum_k \dot{m}_k (h_k + ke_k + pe_k + \dots). \quad (1.1)$$

The term on the left is the rate at which energy is changing within the system. The heat entering or leaving the system is given by \dot{Q}_i and the work entering or leaving the system is given by \dot{W}_j . Material can enter or leave the system by \dot{m}_k that includes enthalpy, h , kinetic and potential energies, ke , pe , etc. In addition, each term is “summed” over an arbitrary number of entry and exit locations i, j, k .

The entropy rate equation for a system [4]

$$\dot{S} = \sum_i \frac{\dot{Q}_i}{T_i} + \sum_k \dot{m}_k s_k + \dot{S}_i = \dot{S}_e + \dot{S}_i. \quad (1.2)$$

Where the left hand term is the rate entropy changes within the system and the right hand terms represent, in order, entropy change due to heat interactions to and from the system and the rate material carries it in or out. These two terms can be combined into one term \dot{S}_e , the entropy exchanged (either positive or negative) with the environment and \dot{S}_i is the irreversible entropy production rate within the system. Figure 1.1 (right) shows the entropy exchanges and production within the system [5].

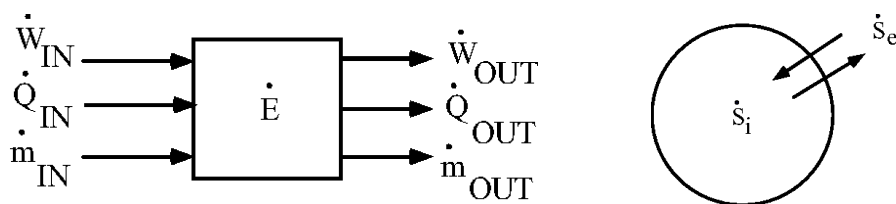


Figure 1.1: Energy flow control volume (left); entropy flux exchange system (right)

The irreversible entropy production rate can be written as the sum of the thermodynamic forces and the thermodynamic flows [5]

$$\dot{S}_i = \sum_k \mathcal{F}_k \dot{\mathcal{X}}_k \geq 0 \quad (1.3)$$

where the entropy change is the sum of all the changes due to the thermodynamic flows \mathcal{X}_k with respect to each corresponding thermodynamic force \mathcal{F}_k .

Next, for systems with a constant environmental temperature (T_o), a thermodynamic quantity called the availability function is defined as [4, 5, 6]

$$\Xi = \mathcal{E} - T_o \mathcal{S}. \quad (1.4)$$

The availability function is described as the maximum theoretically available energy that can do work which we call exergy. Exergy is also known as negative-entropy [4, 6]. Taking the time derivative of the availability function (1.4) and substituting in the expressions for (1.1) and (1.2) results in the exergy rate equation

$$\dot{\Xi} = \sum_i \left(1 - \frac{T_o}{T_i}\right) \dot{Q}_i + \sum_j \left(\dot{W}_j - p_o \frac{d\bar{V}}{dt}\right) + \sum_k \dot{m}_k \zeta_k^{flow} - T_o \dot{\mathcal{S}}_i. \quad (1.5)$$

Where $\dot{\Xi}$ is the rate at which exergy stored within the system is changing. The terms on the right, in order, define the rate exergy is carried in/out by; i) heat, ii) work (less any work the system does on the environment at constant environmental pressure p_o if the system volume \bar{V} changes), and iii) by the material (or quantity known as flow exergy). The final term, $T_o \dot{\mathcal{S}}_i$, is the rate exergy is destroyed within the system or exergy consumption rate.

3 Hamiltonian Mechanics

In this section the basic concepts of Hamiltonian mechanics are introduced. The derivation of the Hamiltonian [7] begins with the Lagrangian for a system defined as

$$\mathcal{L} = \mathcal{T}(q, \dot{q}, t) - \mathcal{V}(q, t) \quad (1.6)$$

where

- t = time explicitly
- q = N-dimensional generalized coordinate vector
- \dot{q} = N-dimensional generalized velocity vector
- \mathcal{T} = Kinetic energy
- \mathcal{V} = Potential energy

The Hamiltonian is defined in terms of the canonical coordinates (q, p) is

$$\mathcal{H}(q, p, t) = \sum_{i=1}^n p_i \dot{q}_i - \mathcal{L}(q, \dot{q}, t) \quad (1.7)$$

where the canonical momentum is defined as

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}. \quad (1.8)$$

Then Hamilton's canonical equations of motion become

$$\begin{aligned}\dot{q}_i &= \frac{\partial \mathcal{H}}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial \mathcal{H}}{\partial q_i} + Q_i\end{aligned}\quad (1.9)$$

where Q_i is the generalized force vector. Next taking the time derivative of (1.7) then substitute (1.9) and simplifying results in the power (work/energy) equation (see ref [8] for details) as

$$\dot{\mathcal{H}}(q, p) = \sum_{i=1}^n Q_i \dot{q}_i. \quad (1.10)$$

4 Thermo-Mechanical Relationships

This section discusses the concepts of conservative systems and forces, reversible and irreversible thermodynamic systems, average power and power flow, and the connections between Hamiltonian mechanics and thermodynamics required to support the concepts of necessary and sufficient conditions for stability of nonlinear systems. It is worth noting at this point that, by definition, electrical power is "pure exergy rate" and the Hamiltonian is stored exergy.

4.1 Conservative Mechanical Systems

A system is conservative if

$$\dot{\mathcal{H}} = 0 \quad \text{and} \quad \mathcal{H} = \text{constant.}$$

A force is conservative if

$$\oint F \cdot dx = \oint F \cdot v dt = \oint Q_j \dot{q}_j dt = 0$$

where F is the force, dx the displacement, and v the velocity. Basically, all of the forces can be modeled as potential force fields which are exergy storage devices.

4.2 Reversible Thermodynamic Systems

A thermodynamic system is reversible if

$$\begin{aligned}d\mathcal{S} &= \frac{dQ}{T} \\ \oint d\mathcal{S} &= \oint \frac{dQ}{T} = 0 \\ \oint d\mathcal{S} &= \oint [d\mathcal{S}_i + d\mathcal{S}_e] = \oint [\dot{\mathcal{S}}_i + \dot{\mathcal{S}}_e] dt = 0\end{aligned}$$

which implies that $\dot{\mathcal{S}}_e = \dot{Q}/T$ since by definition the second law gives $\dot{\mathcal{S}}_i = 0$.

4.3 Irreversible Thermodynamic Systems

For

$$\oint dS = \oint [\dot{S}_i + \dot{S}_e] dt = 0$$

then $\dot{S}_e \leq 0$ and $\dot{S}_i \geq 0$.

4.4 Analogies and Connections

Now the connections between thermodynamics and Hamiltonian mechanics are investigated.

1. The irreversible entropy production rate can be expressed as

$$\dot{S}_i = \sum_k \mathcal{F}_k \dot{\mathcal{X}}_k = \frac{1}{T_o} \sum_k Q_k \dot{q}_k \geq 0. \quad (1.11)$$

2. The time derivative of the Hamiltonian is equivalent to the exergy rate since the Hamiltonian for a conservative system is stored exergy, then

$$\begin{aligned} \dot{\mathcal{H}} &= \sum_k Q_k \dot{q}_k \\ \dot{\Xi} &= \dot{W} - T_o \dot{S}_i = \sum_{j=1}^N Q_j \dot{q}_j - \sum_{l=N+1}^{M+N} Q_l \dot{q}_l. \end{aligned} \quad (1.12)$$

Where N is the number of generators, M the number of dissipators, and let $\dot{W} = \sum_j \dot{W}_j$. The following assumptions apply when utilizing the exergy rate equation (1.5) for *Hamiltonian systems*:

- (a) No substantial heat flow: $\dot{Q}_i \approx 0$.
- (b) No substantial exergy flow or assume T_i is only slightly greater than T_o : $1 - T_o/T_i \approx 0$.
- (c) No $p_o \bar{V}$ work on the environment: $p_o \bar{V} dt = 0$.
- (d) No mass flow rate: $\sum_k \dot{m}_k \zeta_k^{flow} = 0$.
- (e) Then define:

$$\begin{aligned} \dot{W} &\geq 0 && \text{power input/generated} \\ T_o \dot{S}_i &\geq 0 && \text{power dissipated.} \end{aligned}$$

3. A conservative system is equivalent to a reversible system when $\dot{\mathcal{H}} = 0$ and $\dot{S}_e = 0$ then $\dot{S}_i = 0$ and $\dot{W} = 0$.
4. For a system that “appears to be conservative”, but is not reversible is defined as:

$$\begin{aligned} \dot{\mathcal{H}}_{ave} &= 0 && = \frac{1}{\tau} \oint [\dot{W} - T_o \dot{S}_i] dt \\ &= (\dot{W})_{ave} - (T_o \dot{S}_i)_{ave} && = \frac{1}{\tau} \oint [\sum_{j=1}^N Q_j \dot{q}_j - \sum_{l=N+1}^{M+N} Q_l \dot{q}_l] dt \\ &= \text{average power over a cycle} \end{aligned}$$

where τ is the period of the cycle. To be more specific about the average power calculations, the AC power factor [9] provides an excellent example. For the general case of alternating current supplied to a complex impedance the voltage and current differ in phase by an angle θ . Thus

$$\begin{aligned}\dot{W} &= P = Q\dot{q} = v i = \sqrt{2}\bar{v} \cos(\omega t + \theta) \cdot \sqrt{2}\bar{i} \cos \omega t \\ &= \bar{v}\bar{i} [\cos \theta + \cos(2\omega t + \theta)]\end{aligned}$$

where P is power, v is voltage (\bar{v}), i is current (\bar{i}), θ is the phase angle, and ω is the frequency. Integrating over a cycle gives $(\dot{W})_{ave} = \bar{v}\bar{i} \cos \theta$ where for the second term $\oint \cos(2\omega t + \theta) dt = 0$.

This is an important set of conditions that will be used in the next section to find the generalized stability boundary.

5. Finally, the power terms are sorted into three categories:

- (a) $(\dot{W})_{ave}$ - power generators; $(Q_j \dot{q}_j)_{ave} > 0$
- (b) $(T_o \dot{\mathcal{S}}_i)_{ave}$ - power dissipators; $(Q_l \dot{q}_l)_{ave} < 0$
- (c) $(T_o \dot{\mathcal{S}}_{rev})_{ave}$ - reversible/conservative exergy storage terms; $(Q_k \dot{q}_k)_{ave} = 0$.

These three categories are fundamental terms in the following definitions and design procedures.

5 Necessary and Sufficient Conditions for Stability

This section describes the concepts from nonlinear control theory that will be used to assess the balance of exergy flows into a system versus the exergy consumption/destruction (irreversible entropy production) in an open, self-organizing system. The balance of these exergy flows determine a fundamental necessary condition for sustainability of a self-organizing system. Further, the need for continuous flow of exergy into a self-organizing system punctuates the need for predictable, persistent exergy sources.

The Lyapunov function is defined as the total energy (stored exergy by our definition) which for most mechanical systems is equivalent to an appropriate Hamiltonian function

$$V = \mathcal{H} \quad (1.13)$$

which is positive definite. The time derivative is

$$\begin{aligned}\dot{V} &= \dot{\mathcal{H}} = \sum_k Q_k \dot{q}_k = \sum_{j=1}^N Q_j \dot{q}_j - \sum_{l=N+1}^{M+N} Q_l \dot{q}_l \\ &= \dot{W} - T_o \dot{\mathcal{S}}_i.\end{aligned} \quad (1.14)$$

5.1 Stability and Instability Theorems

To describe a nonlinear self-organizing system's behavior two theorems [10] help to characterize the essential features of their motion. In addition, by bounding the Lyapunov function between these Theorems, both necessary and sufficient conditions are a result of the transition of the time derivative of the Lyapunov function from stable to unstable.

1. **Lyapunov Theorem for Stability** Assume that there exists a scalar function V of the state x , with continuous first order derivatives such that

$$\begin{aligned} V(x) & \text{ is positive definite} \\ \dot{V}(x) & \text{ is negative definite} \\ V(x) & \rightarrow \infty \text{ as } \|x\| \rightarrow \infty \end{aligned}$$

Then the equilibrium at the origin is globally asymptotically stable.

2. **Chetaev Theorem for Instability** Considering the equations of disturbed motion, let V be zero on the boundary of a region R which has the origin as a boundary point, and let both V and \dot{V} be positive-definite in R ; then the undisturbed motion is unstable at the origin.

5.2 Stability Lemma for Nonlinear Self-Organizing Systems

Based on the relationships between thermodynamic exergy and Hamiltonian systems a Fundamental Stability Lemma can be formulated.

Fundamental Stability Lemma for Hamiltonian Systems The stability of Hamiltonian systems is bounded between Theorems 1 and 2. Given the Lyapunov derivative as a decomposition and sum of exergy generation rate and exergy dissipation rate (See (1.13) and (1.14)) that is subject to the following general necessary and sufficient conditions:

$$\begin{aligned} T_o \dot{\mathcal{S}}_i & \geq 0 \quad \text{Positive semi-definite, always true} \\ \dot{W} & \geq 0 \quad \text{Positive semi-definite; exergy pumped into the system.} \end{aligned}$$

The following corollaries encompass both stability and instability for Hamiltonian systems which utilize AC power concepts [9]:

Corollary 1: For $(T_o \dot{\mathcal{S}}_i)_{ave} = 0$ and $(\dot{W})_{ave} = 0$ then $\dot{V} = 0$ the Hamiltonian system is neutrally stable, conservative and reversible.

Corollary 2: For $(T_o \dot{\mathcal{S}}_i)_{ave} = 0$ and $(\dot{W})_{ave} > 0$ then $\dot{V} > 0$ the Hamiltonian system is unstable.

Corollary 3: For $(T_o \dot{\mathcal{S}}_i)_{ave} > 0$ and $(\dot{W})_{ave} = 0$ then $\dot{V} < 0$ the Hamiltonian system is asymptotically stable and a passive system in the general sense (passivity controllers).

Corollary 4: Given apriori $(T_o\dot{\mathcal{S}}_i)_{ave} > 0$ and $(\dot{W})_{ave} > 0$ then the Hamiltonian system is further subdivided into:

4.1: For $(T_o\dot{\mathcal{S}}_i)_{ave} > (\dot{W})_{ave}$ with $\dot{V} < 0$ yields asymptotic stability

4.2: For $(T_o\dot{\mathcal{S}}_i)_{ave} = (\dot{W})_{ave}$ with $\dot{V} = 0$ yields neutral stability

4.3: For $(T_o\dot{\mathcal{S}}_i)_{ave} < (\dot{W})_{ave}$ with $\dot{V} > 0$ yields an unstable system.

The bottom line is that stability is defined in terms of power flow which determines whether the system is moving toward or away from its minimum energy and maximum entropy state.

5.3 Classic van der Pol Equation Example

Before moving on to self-organizing systems, it is instructive to provide a couple of simple examples. Example 1 is the classic van der Pol's equation [11] which is analyzed using the techniques of this section. Originally, the "van der Pol equation" is credited to van der Pol, and is a model of an electronic circuit for early radio vacuum tubes of a triode electronic oscillator [11]. The tube acts like a normal resistor when the current is high, but acts as a negative resistor if the current is low. The main feature is that electrical circuits that contain these elements pump up small oscillations due to a negative resistance when currents are small, but drag down large amplitude oscillations due to positive resistance when the currents are large. This behavior is known as a *relaxation oscillation*, as each period of the oscillation consists of a slow buildup of energy ('stress phase') followed by a phase in which energy is discharged ('relaxation phase'). This particular system has played a large role in nonlinear dynamics and has been used to study limit cycles and self-sustained oscillatory phenomena in nonlinear systems.

Consider the van der Pol equation which includes a non-linear damping term:

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0.$$

Next include the actual mass and stiffness values (other than unity) or

$$m\ddot{x} - \mu(1 - x^2)\dot{x} + kx = 0.$$

The appropriate Hamiltonian/Lyapunov function is defined as:

$$\mathcal{H} = V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 > 0.$$

Then the corresponding time derivative of the Lyapunov function becomes

$$\begin{aligned} \dot{V} &= [m\dot{x} + kx]\dot{x} \\ &= [\mu\dot{x}(1 - x^2)]\dot{x} \\ &= \mu\dot{x}^2 - \mu x^2\dot{x}^2. \end{aligned}$$

Identifying generator and dissipator terms yields

$$\begin{aligned}\dot{W} &= \mu \dot{x}^2 \\ T_o \dot{S}_i &= \mu x^2 \dot{x}^2\end{aligned}$$

The stability boundary can be determined as

$$\begin{aligned}\begin{bmatrix} \dot{W} \\ \mu \dot{x}^2 \end{bmatrix}_{ave} &= \begin{bmatrix} T_o \dot{S}_i \\ \mu x^2 \dot{x}^2 \end{bmatrix}_{ave}\end{aligned}$$

By investigating several initial conditions both inside, on, and outside the limit cycle then three separate conditions can be observed. Figure 1.2 shows these conditions with the corresponding numerical values given in Table 1.1.

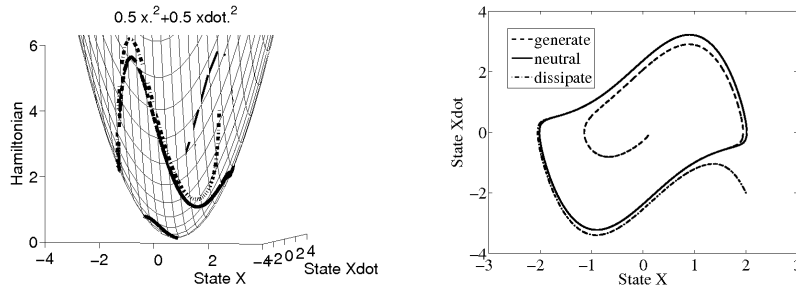


Figure 1.2: Van der Pol responses: Hamiltonian 3D surface (left) and phase plane 2D projection (right)

Table 1.1: Van der Pol model numerical values

Case	x_o	\dot{x}_o	μ	m	k
	(m)	(m/s)	(kg/s)	(kg)	(kg/s ²)
generate	0.1	-0.1	1.5	1.0	1.0
neutral	1.0	-1.0	1.5	1.0	1.0
dissipate	2.0	-2.0	1.5	1.0	1.0

The responses are plotted on the Hamiltonian 3D surface (left) with the projection onto the phase plane shown on the 2D plot (right). For the case outside the limit cycle, the dissipator term dominates and for the case inside the limit cycle the generator term dominates. For both cases inside and outside the limit cycle, the system migrates back to the stability boundary. For the

case already on the limit cycle then the system is already at neutral stability. The neutral exergy-rate (left) and exergy (right) plots are shown in Fig. 1.3. The cycle is defined at approximately $\tau = 3.5$ seconds. For the neutral pair the terms cancel each other out at the end of the cycle or $[\dot{W}]_{ave} = [T_o \dot{S}_i]_{ave}$. For the generator case then $[\dot{W}]_{ave} > [T_o \dot{S}_i]_{ave}$ and for the dissipator case then $[\dot{W}]_{ave} < [T_o \dot{S}_i]_{ave}$, respectively. Eventually, given enough cycles both the generator and dissipator cases will converge to the neutral case.

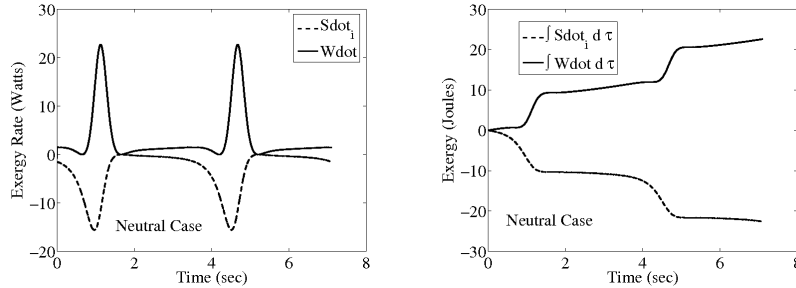


Figure 1.3: Van der Pol exergy-rate (left) and exergy (right) responses - neutral case

Notice that the trajectories are constrained to move along the Hamiltonian surface. By analogy, the Hamiltonian surface enables a “lifestyle” defined by “population” (mass), “investment/infrastructure” (stiffness), “production” (exergy input), and “consumption” (irreversible entropy production). The production and consumption are analogous to supply and demand that are enabled by an infrastructure which supports a population. These concepts will be used in the next section to discuss the sustainability of a lifestyle.

6 Self-Organization and Adaptability Concepts

In this section self-organizing system concepts are discussed that will be used to analyze the sustainability of a simplified nonlinear system model which represents a satellite in space, for example, the earth. The basic format from Heylighen [3] will be followed with support from Haken [12] and Buenstorf [13].

6.1 Background

The Achilles heel or single point of failure of self-organizing systems is the requirement that exergy continuously flow into the system. The self-organizing system is continuously “shedding” entropy to the environment to keep itself organized and living as it consumes or dissipates the exergy flow.

Schrödinger (1945) suggested that all organisms need to import “negative entropy” from their environment and export high entropy (for example, heat) into their environment in order to survive. This idea was developed into a general

thermodynamic concept by Prigogine and his co-workers who coined the notion of “dissipative structures” (Prigogine, 1976; Prigogine and Stengers, 1984), structures of increasing complexity developed by open systems on the basis of energy exchanges with the environment. In the self-organization of dissipative structures, the environment serves both as a source of low-entropic energy and as a sink for the high-entropic energy which is necessarily produced [3].

Basically, self-organizing systems are attempting to balance and perform dialectic synthesis on evolving disordering and ordering pressures [14]. Said another way, life is exergy dissipation (increasing entropy; disorder) and order production in an open system simultaneously. This process which is the evolution of a complex adaptive system is irreversible: the future is fundamentally different from the past, and it is impossible to reconstruct the past from the present [3].

Dissipation is the disordering power flow which is better known as consumption in economics and irreversible entropy production in thermodynamics. Exergy flow into a system is the ordering power flow that is better known as production in economics and exergy rate into an open system in thermodynamics. Balance between these competing power flows is key because these terms are relative to a goal and path through time which means they can “flip over” or reverse roles. For example, the exergy flow into a system by a nuclear weapon is not “matched.” It deposits exergy at a rate that destroys the system, which means it is a disordering power flow increasing entropy. So, a mechanism must be inserted to “match” the input to the system if the goal is sustainability instead of destruction. Nuclear power is an attempt to match the exergy source to the exergy sink to move toward exergy sustainability.

The balance between these opposing power flows creates a sort of “equilibrium condition” for a self-organizing system. Ilya Prigogine described this as “far from thermodynamic equilibrium on the basis of energy dissipation” and, in cybernetics, it’s often called an attractor [3]. Most nonlinear self-organizing systems have several attractors and the system moves between these attractors (reordering) due to variations (perturbations; noise; disorder) in the exergy flow and the system parameters. These system parameters are often called “control parameters” because their values determine the stability characteristics of the system. For example, the potential force field [12, 15] for a nonlinear spring system can be written in kinematic form (no dynamics) as

$$\dot{q} = kq + k_{NL}q^3$$

where k is the linear stiffness coefficient and k_{NL} is the nonlinear stiffness coefficient. The potential function is defined as

$$\mathcal{V}(q) = \frac{1}{2}kq^2 + \frac{1}{4}k_{NL}q^4. \quad (1.15)$$

This system changes its fundamental stability structure by changing $k > 0$ to $k < 0$ and $k_{NL} > 0$. Figure 1.4 shows how the stable equilibrium state at $q = 0$ bifurcates into two symmetrical stable equilibrium states and becomes an unstable state.

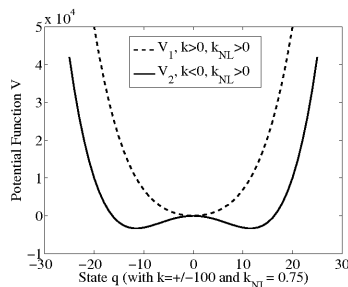


Figure 1.4: Nonlinear spring potential function characteristics

These attractors are defined relative to a “fitness index.” Some attractors are more likely to survive, more fit, than others. In the previous example, one attractor turned into two attractors which appear to be equally fit if the potential function is interpreted as the fitness surface. In fact, it is possible for the system to jump back and forth between these two attractors by varying the exergy flow and the control parameter through perturbations and noise.

As described earlier, nonlinear systems have several attractors and variations or “fluctuations” that reside between attractors in a system that will push the system to one or the other of the attractors. Positive feedback is necessary for random fluctuations to be amplified (generative) [13, 16]. Maintenance of the structured state in the presence of further fluctuations implies that some negative feedback is also present that dampens (dissipates) these effects [13, 16]. In Section 4, these are called a power generator and a power dissipator. Self-organization results from the interplay of positive and negative feedback [16]. In Section 5, this is defined as the stability boundary and/or limit cycle. In more complex self-organizing systems, there will be several interlocking positive and negative feedback loops, so that changes in some directions are amplified while changes in other directions are suppressed [3, 13]. At the transition between order and disorder, a large number of bifurcations may be in existence which are analogous to the bifurcations of the previous potential function. Bifurcations may be arranged in a “cascade” where each branch of the fork itself bifurcates further and further, characteristic of the onset of the chaotic regime [3, 5, 13]. The system’s behavior on this edge is typically governed by a “power law” where large adjustments are possible, but are much less probable than small adjustments [3]. These concepts enable us to better understand nonlinear systems, also known as *complex adaptive systems*, that are on the “edge of chaos” or those systems that are in a domain between frozen constancy (equilibrium) and turbulent, chaotic activity [3]. The mechanism by which complex systems tend to maintain on this critical edge has also been described as *self-organized criticality* [3, 17].

6.2 Simple Nonlinear Satellite System

With this brief background, it's time to analyze a simplified nonlinear satellite system (see Fig. 1.5) to develop the definition of exergy sustainability. For purposes of clarity, each control volume is subdivided into two subregions that contain the physical components. The component mass is constant. A single constant temperature characterizes each component. The component subregion is surrounded by an outer zone that characterizes the interaction between the component (at temperature T) and the environment (reservoir) characterized by temperature T_o .

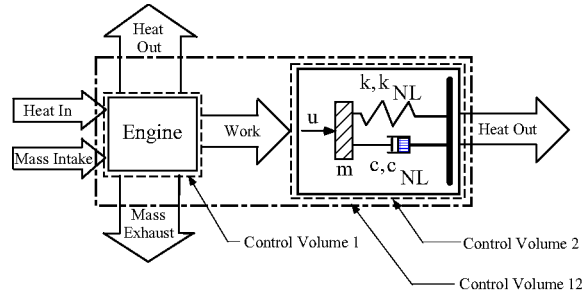


Figure 1.5: Simplified nonlinear satellite model

Conservation Equations for the Machine (Control Volume 2):

Control Volume 2 consists of a nonlinear mass/spring/damper system (with Duffing oscillator/Coulomb friction contributing to the nonlinear effects). Work is supplied to the system which results in the acceleration of the mass. The work is dissipated by the damper. This increases the temperature of the components of the system, which is characterized by a single temperature, T_2 . The thermal energy is then transferred to the environment, which has a temperature of T_o . The transfer is realized solely by a heat interaction. No mass enters or leaves Control Volume 2. Performing the energy, entropy, and exergy analyses [18] yields:

$$\begin{aligned}\dot{\mathcal{E}}_{2_{component}} &= \dot{W} - \dot{Q}_{2_{out}} \\ \dot{\mathcal{S}}_{2_{component}} &= \dot{\mathcal{S}}_{irr2} - \frac{\dot{Q}_{out2}}{T_2} \\ \dot{\Xi}_{2_{component}} &= \dot{W} - T_o \dot{\mathcal{S}}_{irr2} - \left(1 - \frac{T_o}{T_2}\right) \dot{Q}_{out2}\end{aligned}$$

For the surface heat interaction(s) the energy, entropy, and exergy equations are:

$$\begin{aligned}\dot{\mathcal{E}}_{2_{interact-Q}} &= 0 = [\dot{Q}_{out2} - \dot{Q}_{out2}] \\ \dot{\mathcal{S}}_{2_{interact-Q}} &= 0 = \dot{Q}_{out2} \left[\frac{1}{T_2} - \frac{1}{T_o} \right] + \dot{\mathcal{S}}_{irr2_{interact-Q}} \\ \dot{\Xi}_{2_{interact-Q}} &= 0 = \left[\left(1 - \frac{T_o}{T_2}\right) \dot{Q}_{out2} - \left(1 - \frac{T_o}{T_o}\right) \dot{Q}_{out1} \right] - T_o \dot{\mathcal{S}}_{irr2_{interact-Q}} \\ &= \left(1 - \frac{T_o}{T_2}\right) \dot{Q}_{out2} - T_o \dot{\mathcal{S}}_{irr2_{interact-Q}}.\end{aligned}$$

Adding the interaction fluxes to the component fluxes produces the complete equations for Control Volume 2 as:

$$\begin{aligned}\dot{\mathcal{E}}_{2total} &= \dot{W} - \dot{Q}_{out2} \\ \dot{\mathcal{S}}_{2total} &= \dot{\mathcal{S}}_{irr2} - \frac{\dot{Q}_{out2}}{T_2} + \dot{Q}_{out2} \left[\frac{1}{T_2} - \frac{1}{T_o} \right] + \dot{\mathcal{S}}_{irr2interact-Q} \\ \dot{\mathcal{E}}_{2total} &= \dot{W} - T_o \dot{\mathcal{S}}_{irr2} - \left(1 - \frac{T_o}{T_2}\right) \dot{Q}_{out2} + \left(1 - \frac{T_o}{T_2}\right) \dot{Q}_{out2} - T_o \dot{\mathcal{S}}_{irr2interact-Q}\end{aligned}$$

and simplifying yields:

$$\begin{aligned}\dot{\mathcal{E}}_{2total} &= \dot{W} - \dot{Q}_{out2} \\ \dot{\mathcal{S}}_{2total} &= -\frac{\dot{Q}_{out2}}{T_o} + \left[\dot{\mathcal{S}}_{irr2} + \dot{\mathcal{S}}_{irr2interact-Q} \right] \\ \dot{\mathcal{E}}_{2total} &= \dot{W} - T_o \left[\dot{\mathcal{S}}_{irr2} + \dot{\mathcal{S}}_{irr2interact-Q} \right] = \dot{W} - T_o \dot{\mathcal{S}}_i\end{aligned}$$

where

$$\dot{\mathcal{S}}_{irr2interact-Q} = \dot{Q}_{out2} \left[\frac{1}{T_o} - \frac{1}{T_2} \right].$$

By following the derivations of the previous sections, this simplified nonlinear model reduces within Control Volume 2 to

$$\begin{aligned}V &= \mathcal{H} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + \frac{1}{4}k_{NL}x^4 \\ \dot{V} &= \dot{\mathcal{H}} = \left[m\ddot{x} + kx + k_{NL}x^3 \right] \dot{x} \\ &= \dot{W} - T_o \dot{\mathcal{S}}_i \\ &= u\dot{x} - C\dot{x}^2 - C_{NL}\text{sgn}(\dot{x})\dot{x} \\ &= \dot{x} \left[-K_P x - K_I \int_0^t x d\tau - K_D \dot{x} - C\dot{x} - C_{NL}\text{sgn}(\dot{x}) \right]\end{aligned}$$

where

$$\begin{aligned}u &= \text{PID feedback controller (Implemented force input)} \\ &= -K_P x - K_I \int_0^t x d\tau - K_D \dot{x}.\end{aligned}$$

Then the following exergy terms are identified as

$$\begin{aligned}\dot{W} &= \left[-K_I \int_0^t x d\tau \right] \dot{x} \\ T_o \dot{\mathcal{S}}_i &= \left[-K_D \dot{x} - C\dot{x} - C_{NL}\text{sgn}(\dot{x}) \right] \dot{x} \\ T_o \dot{\mathcal{S}}_{rev} &= \left[m\ddot{x} + kx + K_P x + k_{NL}x^3 \right] \dot{x}.\end{aligned}$$

The generalized stability boundary is given as a balance between “positive and negative feedback” (exergy generation and exergy dissipation)

$$\begin{aligned}[\dot{W}]_{ave} &= [T_o \dot{\mathcal{S}}_i]_{ave} \\ \left[-K_I \int_0^t x d\tau \cdot \dot{x} \right]_{ave} &= \left[(K_D + C)\dot{x}^2 + C_{NL}\text{sgn}(\dot{x}) \cdot \dot{x} \right]_{ave}.\end{aligned}\quad (1.16)$$

The “shape” of the resulting limit cycle is constrained to the Hamiltonian surface which determines the accessible bifurcated structure as a function of exergy level (see Figs. 1.4).

6.3 Lifestyle Definition

Next this model is interpreted in terms of a “lifestyle” of the mass-spring-damper system within the satellite. First, the lifestyle is defined by a cyclic path, attractor, or limit cycle in the phase plane that is constrained to the Hamiltonian surface $\mathcal{H} = V$, (left) projected onto the phase plane (right) in Fig. 1.6. This path occurs as a result of satisfying (1.16). This interpretation directly provides the definition of exergy sustainability: the continuous compensation of irreversible entropy production in an open system with an impedance and capacity-matched persistent exergy source. In other words, the cyclic lifestyle will persist indefinitely as long as (1.16) is satisfied and m, k, K_P , and k_{NL} are constants.

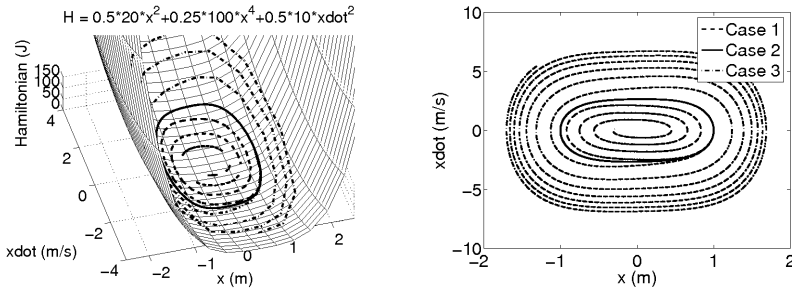


Figure 1.6: All cases: mass-spring-damper with Duffing oscillator/Coulomb friction model numerical results: Hamiltonian 3D surface (left) and phase plane 2D projection (right)

Second, this lifestyle will change if the “population” (mass), “investment/infrastructure” (stiffness), “production” (exergy input), and/or “consumption” (irreversible entropy production) are changed independently because all of these parameters are interconnected through the system. Since the lifestyle path over time is constrained to the Hamiltonian surface, if the population is changed then the stiffness can be changed to hold the lifestyle constant while holding production and consumption constant. The infrastructure is expanded to accommodate the increasing population. On the other hand, if the population increases then the consumption increases which means the production must increase as well. In addition, if the lifestyle increases by creating more services (more exergy consumption) then the production must increase to offset consumption which is enabled through an expanded infrastructure. Bottom-line: lifestyle is directly related to exergy consumption and how exergy sources are matched to exergy sinks through the infrastructure.

Third, this simplified lifestyle presents the mass-spring-damper system as an exergy parasite on the satellite with respect to the sun. By digging a little deeper into the model, the parasites become humans on the earth with respect to the sun. The goal of exergy sustainability now includes striving to become a symbiotic versus a destructive parasite, such as the GAIA approach [19], because

cleaning up the disordering effects of a destructive parasite will only consume more exergy through an impedance mismatch. In fact, the human race has done an impressive job of overcoming the impedance mismatch between population growth and the carrying capacity of the biosphere with fossil fuels; we effectively eat fossil fuel.

One final observation on this topic of impedance and capacity matching, the goal of war and economic competition is to create a production/consumption rate which is sustainable for you and generates an impedance mismatch that is unsustainable for your enemy/competitor. The ultimate goal is to cut-off, destroy, and/or dissipate your competitor's exergy reserves by changing/deforming your competitor's Hamiltonian surface (infrastructure - including population of the work force). This can be accomplished in several ways including: 1) pick-up the pace by increasing the limit cycle frequency (i.e., less mass), 2) accelerate the exergy consumption of your competitor by using more efficient technologies, and 3) deform the potential field with information flow.

7 Summary and Conclusions

This paper has developed a new definition of exergy sustainability: the continuous compensation of irreversible entropy production in an open system with an impedance and capacity-matched persistent exergy source. The development of this definition has led to a missing link in the analysis of self-organizing systems: a tie between irreversible thermodynamics and Hamiltonian systems. This tie was exploited through nonlinear control theory to define necessary and sufficient conditions for stability of nonlinear systems that were employed to formulate the concept of "on the edge of chaos." Finally, an equivalence was developed between physical stored exergy and information-based exergy which can be exploited to change a lifestyle and enhance one's economic competitiveness or performance on the battlefield.

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