Interconnecting Robotic Subsystems in a Network

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1. Abstract

This paper analyzes how the interconnection of subsystems affects the network dynamics of the overall system's connective stability. Some closed form equations for the magnitude of the interaction between subsystems are obtained to aid in the design of nearest neighbor interconnected subsystems or fully interconnected subsystems.

2. Introduction

Multi robot formation algorithms have been of considerable importance in the control and robotics communities due to their applications in search and rescue missions, air traffic control, automatic highways and military operations such as reconnaissance, surveillance and target acquisition to mention a few. The analysis of connective stability for different subsystem interconnections in robot formations has not been examined, and so, this paper is intended to fill that gap.

In previous work multi robot formations have been accomplished in several different ways. Behavioral-based approaches have been used for robot formations [Balch 1998], [Arkin 1992] and [Kube 1993], but do not include a formal development of the system controls from a stability point of view. Artificial formations have been studied using analogs to animal behavior. Reynolds developed a simple egocentric behavioral model for flocking which is instantiated in each member of the simulated group of birds. The behavior consists of several separate components [Reynolds 1987]. In other studies algorithms for formations have been proposed such as in [Fredslund 2001] and [Fredslund 2002], where each robot keeps a single *friend* at a desired angle

and the goal becomes to center the friend in the robot sensor's field of view. More recent work has been taking a system control perspective. Feddema [Feddema 2001], [Feddema 2002] uses decentralized control theory to control motion of multiple robotic vehicles in formation along a line. Desai [Desai 1998], [Desai 2001] modeled a formation of nonholonomic mobile robots and developed a framework for transitioning from one formation to another. Chen and Luh [Chen 1994] used distributed control in large groups of robots to cooperatively move in various geometric formations.
In this paper, the preliminaries for decentralized control of large scale systems [Siljak 1991] and [Siljak 1990] are introduced, and the connective stability analysis of multiple subsystems using Lyapunov vector functions are addressed. Three types of network interconnections are then defined. Part 6 defines the state space representation for the robot dynamics, which then proceeds to a complete definition of the parameters for two subsystems of interest: a *diagonal subsystem* and the *robot subsystem*. In part 8, the results for connective stability are computed, along with some closed form equations that are useful when designing nearest neighbor and fully interconnected subsystems.

3. Preliminaries

Let us consider a dynamic system *S* described by the differential equation,

$$S: \quad \begin{aligned} \dot{\vec{x}} &= f(t,\vec{x},\vec{u}) \\ \vec{y} &= h(t,\vec{x}) \end{aligned} \qquad (1)$$

where $\vec{x}(t) \in \Re^n$ is the state of S at time $t \in \Im$, $\vec{u}(t) \in \Re^p$ are the inputs and $\vec{y}(t) \in \Re^q$ are the outputs. Assume the function $f: \Im \times \Re^n \times \Re^p \to \Re^n$ (which describes the dynamics of S) to be defined and continuous on the domain $\Im \times \Re^n \times \Re^p$. Assume the function $h: \Im \times \Re^n \to \Re^q$ (which describes the observations of S) to be defined and continuous on the domain $\Im \times \Re^n$, so that solutions of (1) exists for all initial conditions. The system S described by (1) can be decomposed into N interconnected subsystems S_i described by the equations,

$$S: \quad \begin{aligned} \dot{\vec{x}} &= f_i(t,\vec{x}_i,\vec{u}_i) + \tilde{f}_i(t,\vec{x},\vec{u}) \\ \vec{y} &= h_i(t,\vec{x}_i) + \tilde{h}_i(t,\vec{x}) \end{aligned} \quad . \qquad (2)$$

In this new description of the system S, the functions $f_i: \Im \times \Re^{n_i} \times \Re^{p_i} \to \Re^{n_i}$ represent the dynamics of each isolated subsystem S_i, and $\tilde{f}_i: \Im \times \Re^n \times \Re^p \to \Re^{n_i}$ describes the dynamic interaction of S_i with the rest of the system S. The function $h_i: \Im \times \Re^{n_i} \to \Re^{q_i}$ represents the observations at S_i from its local state variables, and the function $\tilde{h}_i: \Im \times \Re^n \to \Re^{q_i}$ represents the observations at S_i from the rest of the system S. Feedback may be added to the system S as follows,

$$\vec{u}_i = k_i(t,\vec{y}_i) + \tilde{k}_i(t,\vec{y}), \quad i \in \{1,..., N\}, \qquad (3)$$

where $k_i: \Im \times \Re^{q_i} \to \Re^{n_i}$ represents the control law applied at S_i from its local state variables, and the function $\tilde{k}_i: \Im \times \Re^q \to \Re^{n_i}$ represents the observations at S_i from the rest of the system S.

For linear time invariant lumped systems, (1) can be described by a set of equations of the form,

$$S: \quad \dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} \tag{4}$$

where A, B and C are, respectively, $n \times n$, $n \times p$, and $q \times n$, constant matrices. The system S in (4) has p inputs, q outputs and n state variables. In a similar way as in (2), the system S described by (4) can be decomposed into N interconnected subsystems S_i described by the equations,

$$S: \quad \dot{\vec{x}}_i = A_i\vec{x}_i + \sum_{j=1}^{N} \bar{e}_{ij} A_{ij} \vec{x}_j + B_i\vec{u}_i + \sum_{j=1}^{N} \bar{b}_{ij} B_{ij} \vec{u}_j, \quad i \in \{1,..., N\},$$
$$\vec{y}_i = C_i\vec{x}_i + \sum_{j=1}^{N} \bar{c}_{ij} C_{ij} \vec{x}_j, \tag{5}$$

where A_i, B_i and C_i are, respectively, $n_i \times n_i$, $n_i \times p_i$, and $q_i \times n_i$, constant matrices. Each subsystem S_i in (5) has p_i inputs, q_i outputs and n_i state variables such that $p = \sum_{i=1}^{N} p_i$, $q = \sum_{i=1}^{N} q_i$ and $n = \sum_{i=1}^{N} n_i$. The matrices A_{ij}, B_{ij} and C_{ij} represent the interaction among the subsystems. The matrices A_i, B_i and C_i represent the "self-interaction" within the same subsystem S_i. The elements $\bar{e}_{ij}$ are defined as

$$\bar{e}_{ij} = \begin{cases} 1, & S_j \quad can\ act\ on \quad S_i \\ 0, & S_j \quad cannot\ act\ on \quad S_i. \end{cases} \tag{6}$$

The matrix $E = (\bar{e}_{ij})$ is called the *fundamental interconnection matrix* [Siljak 1990]. The elements $\bar{b}_{ij}$ are defined as,

$$\bar{b}_{ij} = \begin{cases} 1, & \vec{u}_j \quad can\ act\ on \quad \dot{\vec{x}}_i, \\ 0, & \vec{u}_j \quad cannot\ act\ on \quad \dot{\vec{x}}_i, \end{cases} \tag{7}$$

and the elements $\bar{c}_{ij}$ are defined as,

$$\bar{c}_{ij} = \begin{cases} 1, & \vec{x}_j \quad can\ be\ observed\ on \quad \vec{y}_i, \\ 0, & \vec{x}_j \quad cannot\ be\ observed\ on \quad \vec{y}_i. \end{cases} \tag{8}$$

A decentralized control can be implemented on the partitioned dynamic system using local control based on local observations. Formally for each subsystem S_i a linear local control of the form,

$$\vec{u}_i = \vec{r}_i - K_i^T \vec{x}_i \tag{9}$$

is applied based on local observations, i.e. each element of the matrix $B_{ij} = 0$ and $C_{ij} = 0 \quad \forall i, j \in \{1,..., N\}$. Substitution of the decentralized control given by (9) in (5) would reconstruct the partitioned closed loop dynamics as,

$$S: \quad \begin{aligned} \dot{\vec{x}}_i &= (A_{CL})_i \vec{x}_i + \sum_{j=1}^{N} \bar{e}_{ij} A_{ij} \vec{x}_j + (B_{CL})_i \vec{r}_i, \quad i \in \{1,..., N\}, \\ \vec{y}_i &= (C_{CL})_i \vec{x}_i, \end{aligned} \tag{10}$$

where $(A_{CL})_i = A_i - B_i K_i^T$, $(B_{CL})_i = B_i$ and $(C_{CL})_i = C_i$ are the closed loop matrices for each subsystem S_i.

4. Connective Stability

Analysis of connective stability is based upon the concept of vector Liapunov functions. It should be mentioned that there exists no general systematic procedure for choosing vector Liapunov functions. The outline method for stability analysis first assumes that the complete system S has been decomposed into N interconnected subsystems S_i. To have stability of each subsystem S_i, the scalar function $v_i(\vec{x}_i) = (\vec{x}_i^T H_i \vec{x}_i)^{1/2}$ is proposed as candidates for the Liapunov functions of each S_i, and require that for any choice of the positive definite matrices G_i, there exist positive definite matrices H_i as solutions of the Liapunov matrix equations $A_i^T H_i + H_i A_i = -G_i$. Now the vector Liapunov function is defined as $\vec{v} = (v_1, v_2, ..., v_N)$, which must satisfy the inequality $\dot{\vec{v}} \le W \vec{v}$ to guaranty connective stability of the overall system S. The elements of the "aggregation matrix" $W = (w_{ij})$ are defined by [Siljak 1991],

$$w_{ij} = \begin{cases} \dfrac{\lambda_m(G_i)}{2\lambda_M(H_i)} - \bar{e}_{ii} \lambda_M^{1/2}\left(A_{ii}^T A_{ii}\right), & i = j \\ -\bar{e}_{ij} \lambda_M^{1/2}\left(A_{ij}^T A_{ij}\right), & i \neq j \end{cases} \tag{11}$$

where $\lambda_m(\bullet)$ and $\lambda_M(\bullet)$ are the minimum and maximum eigenvalues of the corresponding matrices. The matrix W must have all leading principal minors positive to guaranty stability of the overall system S. This method is very powerful, because it guaranties that the system S will be stable even if an interconnection becomes decoupled, i.e. $\bar{e}_{ij} = 0$, or if the interconnection parameters are perturbed, i.e. $0 < \bar{e}_{ij} < 1$ [Siljak 1990].

5. Network Interconnection types

The following interconnections for a team of independent subsystems (robots) are considered (Fig. 1),

1) *Nearest neighbor interconnection (NN)*: each subsystem interacts only with its nearest neighbors.

2) *Nearest neighbor interconnection with a central unit (NC)*: each subsystem interacts with its nearest neighbors and with a central unit (subsystem 1).

3) *Full interconnection (FI)*: every subsystem interacts with every other subsystem.

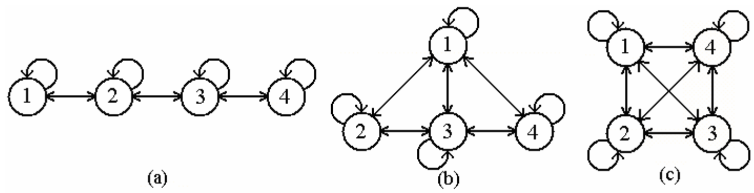

Figure. 1. Formations for four subsystems (robots): (a) nearest neighbor interconnection, (b) nearest neighbor interconnection with a central unit (1), and (c) full interconnection.

The dynamics of the overall system S for the nearest neighbor interconnection for N subsystems can be described by (Fig. 2),

$$\begin{aligned}
\dot{\vec{x}}_1 &= \left(A_1 - B_1K_1^T\right)\vec{x}_1 + B_1Q\left(C_2\vec{x}_2 - C_1\vec{x}_1\right) + B_1\vec{r}_1 \\
\dot{\vec{x}}_2 &= \left(A_2 - B_2K_2^T\right)\vec{x}_2 + B_2Q\left(C_1\vec{x}_1 - C_2\vec{x}_2\right) + B_2Q\left(C_3\vec{x}_3 - C_2\vec{x}_2\right) + B_2\vec{r}_2 \\
\dot{\vec{x}}_3 &= \left(A_3 - B_3K_3^T\right)\vec{x}_3 + B_3Q\left(C_2\vec{x}_2 - C_3\vec{x}_3\right) + B_3Q\left(C_4\vec{x}_4 - C_3\vec{x}_3\right) + B_3\vec{r}_3 \\
&\vdots \\
\dot{\vec{x}}_{N-1} &= \left(A_{N-1} - B_{N-1}K_{N-1}^T\right)\vec{x}_{N-1} + B_{N-1}Q\left(C_{N-2}\vec{x}_{N-2} - C_{N-1}\vec{x}_{N-1}\right) + B_{N-1}Q\left(C_N\vec{x}_N - C_{N-1}\vec{x}_{N-1}\right) + B_{N-1}\vec{r}_{N-1} \\
\dot{\vec{x}}_N &= \left(A_N - B_NK_N^T\right)\vec{x}_N + B_NQ\left(C_{N-1}\vec{x}_{N-1} - C_N\vec{x}_N\right) + B_N\vec{r}_N
\end{aligned} \tag{12}$$

where Q is the interaction gain matrix between subsystems. The fundamental interconnection matrix that describes the interactions of (12) is given by the $N \times N$ constant matrix,

$$\overline{E} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 1 & 0 & & 0 \\ 0 & 1 & 1 & 1 & & 0 \\ \vdots & & & & \ddots & \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{13}$$

Note that $A_{11} = -B_1QC_1$, $A_{12} = B_1QC_2$, etc.

In a similar way, the fundamental interconnection matrices for the nearest neighbor interconnection, with subsystem 1 as the central unit, and the full interconnection interaction, can be computed as the following $N \times N$ constant matrices,

$$\overline{E} = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & 0 & & 0 \\ 1 & 1 & 1 & 1 & & 0 \\ \vdots & & & & \ddots & \\ 1 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}, \tag{14}$$

and

$$\overline{E} = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & 1 & & 1 \\ 1 & 1 & 1 & 1 & & 1 \\ \vdots & & & & \ddots & \\ 1 & 1 & 1 & 1 & \cdots & 1 \end{bmatrix}, \tag{15}$$

respectively.

6. Robot dynamics

In a group formation of mobile robots a natural way to select the subsystems is

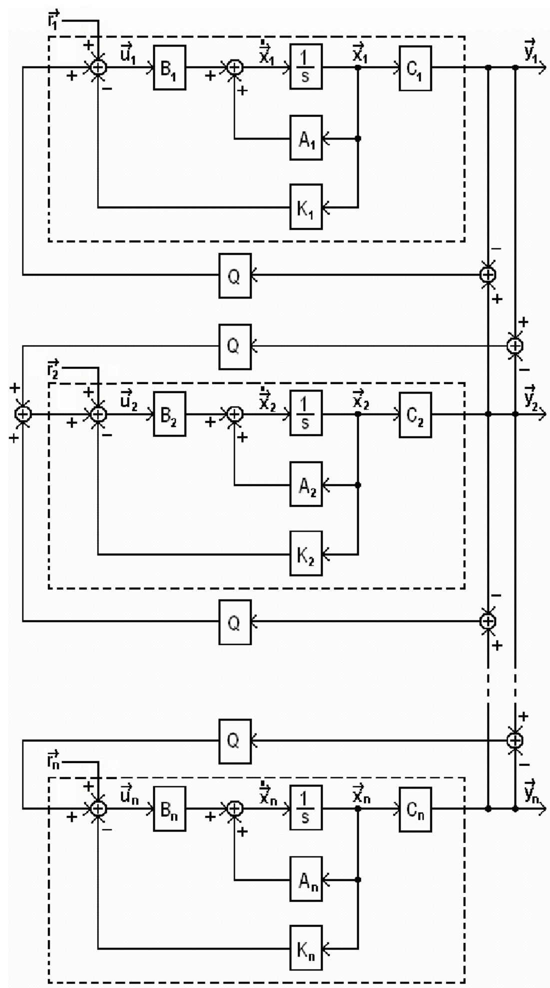

Figure. 2. Control block diagram for the nearest neighbor interconnection of N subsystems.

selecting each robot as a subsystem. The equations of motion for each robot (subsystem) are based on simple Newtonian point physics [Cliff 1996], and a model that captures the torque produced by a direct current (DC) motor given by [Kalmar 2004],

$$\tau = \alpha V - \mu\omega, \tag{16}$$

where V is the voltage applied to the DC motor, ω is the angular velocity of the motor shaft and (α, μ) are constants properties of the DC motor given by $\alpha = k_I / R_a$ and $\mu = k_f k_I / R_a$, where k_I is the motor torque constant, k_f is the back-emf constant of the motor and R_a is the armature resistance.

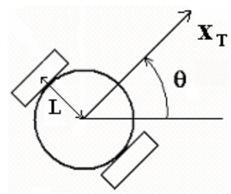

Figure. 3. Differential drive robot. Subsystem dynamics for each robotic vehicle.

For a two wheeled drive mobile robot (Fig. 3), the equation of motion that describes its translation x_T is given by,

$$m\frac{d^2 x_T}{dt^2} + b_{eff}\frac{dx_T}{dt} = \alpha_R V_R + \alpha_L V_L, \tag{17}$$

where m is the mass of the robot, b_{eff} is the effective viscous damping coefficient, and V_R and V_L are the voltages being applied into the right and left motors that drive the corresponding wheels. α_R and α_L are the constants for the right and left motors. It can be shown that $b_{eff} = b + (\mu_R + \mu_L)/r_\omega^2$, where r_ω is the radius of the wheel, and b is the friction coefficient of the vehicle in the media.

The equation of motion that describes its changes in orientation θ is given by,

$$J\frac{d^2\theta}{dt^2} + b_{\theta eff}\frac{d\theta}{dt} = \frac{L}{r_\omega}\alpha_R V_R - \frac{L}{r_\omega}\alpha_L V_L, \tag{18}$$

where J is the moment of inertia, $b_{\theta eff}$ is the effective angular viscous damping, and L is the distance from the wheel to the center of the robot (Fig. 3). It can be shown that $b_{\theta eff} = b_\theta + L^2(\mu_R - \mu_L)/r_\omega^2$, where b_θ is the angular friction coefficient. Defining the state vector as $\vec{x} = (x_1, x_2) = (\dot{x}_T, \dot{\theta})$, (17) and (18) are a two input two output system that can be written in state space as,

$$S_R: \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} -\frac{b_{eff}}{m} & 0 \\ 0 & -\frac{b_{\theta eff}}{J}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{m}\alpha_R & \frac{1}{m}\alpha_L \\ \frac{L}{Jr_\omega}\alpha_R & -\frac{L}{Jr_\omega}\alpha_L\end{bmatrix}\begin{bmatrix}V_R \\ V_L\end{bmatrix}, \tag{19}$$

and,

$$S_R: \begin{bmatrix}y_1 \\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}. \tag{20}$$

7. Subsystems

The following two subsystems are of particular interest,

Any diagonal subsystem S_D

It is assumed that every single subsystem S_i is represented by a $n \times n$ state space with n inputs and n outputs. Stability of each subsystem is accomplished by using pole placement such that $A_i - B_i K_i^T = -\beta I_i$ where $\beta > 0$ and I_i is the identity $n \times n$ matrix. Assume each subsystem to be fully controllable and observable by setting

$B_i = I_i$ and $C_i = I_i$ for all $i = \{1,...,N\}$ where I_i is the identity $n \times n$ matrix. The interaction gain matrix between subsystems is given by $Q = \beta\gamma I_i$ where $\gamma > 0$ and I_i is the identity $n_i \times n_i$ matrix. For the nearest neighbor interconnection, the dynamics of the overall system S for N subsystems described by (11) would be given as,

$$S_D(NN): \begin{aligned} \dot{\vec{x}}_1 &= -\beta(1+\gamma)\vec{x}_1 + \beta\gamma\vec{x}_2 + \vec{r}_1 \\ \dot{\vec{x}}_2 &= -\beta(1+2\gamma)\vec{x}_2 + \beta\gamma(\vec{x}_1 + \vec{x}_3) + \vec{r}_2 \\ \dot{\vec{x}}_3 &= -\beta(1+2\gamma)\vec{x}_3 + \beta\gamma(\vec{x}_2 + \vec{x}_4) + \vec{r}_3 \\ &\vdots \\ \dot{\vec{x}}_{N-1} &= -\beta(1+2\gamma)\vec{x}_{N-1} + \beta\gamma(\vec{x}_{N-2} + \vec{x}_N) + \vec{r}_{N-1} \\ \dot{\vec{x}}_N &= -\beta(1+\gamma)\vec{x}_N + \beta\gamma\vec{x}_{N-1} + \vec{r}_N. \end{aligned} \quad (21)$$

Robotic subsystem S_R

Each subsystem S_i has the state space representation given by (19) and (20). The dynamics of the overall system S, for the nearest neighbor interconnection for N subsystems, is described by (12). The following parameters have been used in this robotic model: $K_I = 2.5283\times10^{-3}\left(\frac{Nm}{A}\right)$, $K_f = 0.055\left(\frac{Vs}{rad}\right)$, $R_a = 0.2(\Omega)$, $b = 1\left(\frac{N}{m/s}\right)$, $b_\theta = 1\left(\frac{Nm}{rad/s}\right)$, $m = 1(Kg)$, $r_w = 0.0325(m)$, $L = 0.055(m)$, $\alpha_L = \alpha_R$, $\mu_L = \mu_R$, $J = 1\left(\frac{Nm}{rad/s^2}\right)$, the full state feedback $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and the interconnection gain matrix $Q = \begin{bmatrix} \gamma & 0 \\ 0 & -\gamma \end{bmatrix}$.

8. Results

The computation of connective stability is done for (a) the diagonal subsystem, and (b) the robotic subsystem. The following results used $\beta = 1$ for the diagonal subsystem. Connective stability is investigated using the "aggregation matrix" define by (11). This test sets an upper bound on the magnitude of the interactions between subsystems. Table (I) and (II) show the maximum magnitude of γ to guaranty connective stability for different subsystem interconnections, where each subsystem used the diagonal model S_D and the robotic model S_R respectively. Define γ_2 as the upper bound on the magnitude of the interactions between 2 subsystems (e.g. $\gamma_2 = 1/2$ in Table (I)), and define q_N as the upper bound on the magnitude of the interactions for N subsystems (e.g. $\gamma < q_6$ for 6 subsystems). As the number of subsystems are increased, q_N decreases with no limit for the *NC* and *FI* interconnections. Only the nearest neighbor interconnection reaches a limit. The limit can be shown to be,

$$\lim_{N \to \infty} q_N = \frac{\gamma_2}{2}. \quad (22)$$

Equation (22) can be used when designing nearest neighbor interconnected subsystems to guaranty connective stability.

Another useful equation can be found for fully interconnected subsystems, by induction,

$$q_N = \frac{\gamma_2}{N-1} \tag{23}$$

gives the upper bound for the interactions when fully connecting *N* subsystems.

9. Conclusion

Some design equations were presented that could assist in the design of nearest neighbor interconnected subsystems or fully interconnected subsystems to guaranty connective stability. The more interconnections in the network, the less connectively stable the overall system becomes.

TABLE I
CONNECTIVE STABILITY FOR DIFERENT SUBSYTEM INTERCONNECTIONS (DIAGONAL SUBSYTEM)

Number of Subsystems	Connective Stability		
N	NN	NC	FI
2	$\gamma < 1/2$	$\gamma < 1/2$	$\gamma < 1/2$
3	$\gamma < 1/3$	$\gamma < 1/4$	$\gamma < 1/4$
4	$\gamma < 0.2928$	$\gamma < 0.1909$	$\gamma < 1/6$
5	$\gamma < 0.2763$	$\gamma < 0.1632$	$\gamma < 1/8$
6	$\gamma < 0.2679$	$\gamma < 0.1437$	$\gamma < 1/10$
7	$\gamma < 0.2630$	$\gamma < 0.1280$	$\gamma < 1/12$
8	$\gamma < 0.2598$	$\gamma < 0.1150$	$\gamma < 1/14$
9	$\gamma < 0.2577$	$\gamma < 0.1041$	$\gamma < 1/16$
10	$\gamma < 0.2562$	$\gamma < 0.0949$	$\gamma < 1/18$
N	NCFK	NCFK	$\gamma < \frac{1/2}{N-1}$
$\lim_{N \to \infty}$	$\gamma < \frac{1/2}{2}$	$\gamma \to 0$	$\gamma \to 0$

NCFK = No closed form known.

TABLE II
CONNECTIVE STABILITY FOR DIFERENT SUBSYTEM INTERCONNECTIONS (ROBOTIC SUBSYTEM)

Number of Subsystems	Connective Stability		
N	NN	NC	FI
2	$\gamma < 16525$	$\gamma < 16525$	$\gamma < 16525$
3	$\gamma < 11017$	$\gamma < 8262$	$\gamma < 8262$
4	$\gamma < 9681$	$\gamma < 6312$	$\gamma < 5508$
5	$\gamma < 9135$	$\gamma < 5396$	$\gamma < 4131$
6	$\gamma < 8855$	$\gamma < 4750$	$\gamma < 3305$
7	$\gamma < 8693$	$\gamma < 4241$	$\gamma < 2754$
8	$\gamma < 8589$	$\gamma < 3802$	$\gamma < 2360$
9	$\gamma < 8519$	$\gamma < 3442$	$\gamma < 2065$
10	$\gamma < 8470$	$\gamma < 3139$	$\gamma < 1836$
N	NCFK	NCFK	$\gamma < \frac{16525}{N-1}$
$\lim_{N \to \infty}$	$\gamma < \frac{16525}{2}$	$\gamma \to 0$	$\gamma \to 0$

NCFK = No closed form known.

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