Eigen Analysis of Model Based Residual Spectra for Fault Diagnostics Techniques

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The energy distribution of Residual Spectra generated by Model Based Parity Space relationship drifts in the form of Eigen value. The Eigen vector in such a multi-dimensional Residual Space is used to maintain the degree and polarity of drift. This paper presents investigations into the issues related to such Eigen analysis. It was found that normalized residuals from multiple sources and parity space relations are neutralized in the form of unified representation of energy that can be used to form a generic framework for fault detection and isolation. The case studies into multi-sensor system were carried out and actual data from sensors is used to test this generic framework. It is being investigated, how the proper modeling of qualitative entities as energy, could lead to unified and neutral residual space while keeping the implementation cost reasonably low.

Key words: Fault Diagnostics, Single Throw Mechanical Equipment (STME), Parameter Estimation, Eigen Values of Faults, Residual generation, Parity-Space, Low-cost limit sensors, Adaptive thresholds.

Introduction

This research builds on the work [i, ii, iii, iv, v] carried out in the area of Single Throw Mechanical Equipment (STME). STME was formally defined as a system that has two stable states. Whenever activated, it physically moves from one state to another. This process of state transition is defined as a throw. It is bounded between two stable states, which in the practical sense relates to two physical barriers placed at a distance `d'. Transition from state A to state B is the forward throw while from state B to state A is the reverse throw. Practically, the time between two throws (forward or reverse) or two operations is dependent on the use and type of equipment and may vary from a couple of throws per minute to once a day. From an operational perspective, it is acceptable to assume that the STME state of health remains same until the next throw is performed.

Initially, the focus of research was only to the STME, but later it was found that such approach can be extended to most of the complex systems in industry where the discrete inputs using switches is only available. Such systems are already being used or can be a cost effective addition. In this paper, the focus has been maintained only to STMEs that can be easily extended to include numerous industrial applications for condition monitoring. For industry [vi, vii], if the new sensors need to be employed, they must have,

- Reliability better than the system itself;
- Cost-effectiveness;
- Minimum maintainability such as calibration;
- Suitability for harsh environment;
- Robustness to noise and disturbances under varying operating conditions.

Limits sensors are the best candidate although because of their limitation to detect a continuous change in residuals, the research and academia maintains focus on continuous sensors, while industry keeps the limit switches as their key sensor. The paper attempts to bridge the gulf and formally presents a framework for deploying optimal number of limit switches to capture the process dynamics and use them in model based residual generation.

FDI Framework for STME

The concept behind the design of such frame work is that the energy from the source or the input is distributed based on the energy paths provided by the system that can be viewed as the eigen values or spectral decomposition of any system. The energy distribution model if perturbs or deviates from the nominal distribution can be detected by the parity space residuals. The basic residual generator [viii, ix] in its time domain representation is,

$$r(t) = V(\phi)u(t) + W(\phi)y(t) \tag{1}$$

For a no-fault case we have r(t)=0 and y(t)=M(ϕ)u(t),

$$\begin{aligned} 0 &= V(\phi)u(t) + W(\phi)M(\phi)u(t) = V(\phi) + W(\phi)M(\phi) \\ V(\phi) &= -W(\phi)M(\phi) \\ r(t) &= W(\phi)[y(t) - M(\phi)u(t)] \end{aligned} \tag{2}$$

The primary residuals o(t) are,

$$o(t) = y(t) - M(\phi)u(t) \tag{3}$$

After substituting *y(t)*,

$$r(t) = W(\phi)\left[S_F(\phi)p(t) + S_D(\phi)q(t) + S_N(\phi)v(t) + \Psi_F(t)\Delta\theta_F + \Psi_D(t)\Delta\theta_D\right] \tag{4}$$

It is now evident that the residual generation process relates primarily to evaluating W(ϕ) which meets the design specifications. This shows that generalised residual generator and parity space residual generator are mathematically similar.

The case can be considered for structured or un-structured residuals. In case of structured residuals, a limited number of faults can be diagnosed neatly based of the basis set provided by the eigen vectors. The energy variations are the measure of the fault or deviation in a desired canonical form and this is limited by the number of independent residuals. For n number of residuals, we can have 2^n uniquely identifiable faults[x]. This results in a special matrix structure representing the effect of faults and disturbances on the residuals.

$$o(t) = \begin{bmatrix} o_1(t) \\ o_2(t) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 1 & 0 \\ m_{21} & m_{22} & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta u_1(t) & \Delta u_2(t) & \Delta y_1(t) & \Delta y_2(t) \end{bmatrix} \quad (5)$$

For a square S_F without any rank defect, complete response can be specified, otherwise only partial specification can be obtained,

$$w_1 = [0 \quad \mathrm{bu}_1] \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}^{-1} \quad (6)$$

$$w_2 = [\mathrm{bu}_2 \quad 0] \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}^{-1}$$

The desired response may now be computed. This may also be achieved using elimination of variables.

The aim of this approach is to generate a number of independent residuals by exploiting the model non-linearities that are left un-modelled when linear models of complex systems are identified. The leftover structure in the residuals has been found sensitive to faults [xi], while significant effect is not observed in controlling such systems. The non-linear aspects become more dominant in case of faults and such residuals play a pivitol role in sensitive residual generator design. The static transformations considered in this case are not static in the sense of process dynamics.

In order to understand this approach, the framework for STME is discussed here. It attempts to utilise the residuals generated from the array of limit switches or position sensors, placed at the boundaries of the linear regions of the observable dynamics profile. The residuals are used to generate an appropriate structure for sensor and system fault isolation. The generic features of this approach are,

- Set of limit-switches used to acquire velocity measurement (ds/dt). Since Δs is fixed and Δt is measured from the limit switches, the velocity can be measured in the required region with reasonable accuracy,
- The Residual Generator prepares a residual-vector with each component sensitive to certain faults and insensitive to others. This is achieved by elimination of variables,
- The Fault Detection and isolation is achieved by using the thresholds computed from the statistics of the structured residuals and are then adapted using exponential models according to their sensitivity at the current operating point.

Cost-effective FDI Sensor

In order to encourage the deployment of FDI in mass production of common use products, it is important to consider the cost of sensors. The recent drop in the prices of micro-electronics has made the processors and Application Specific Integrated Circuits (ASIC) very cheap, but no significant drop in sensors prices has been observed for industrial use. Cost of limit detectors is very low as compared to other similar sensors. The initial approach employed sensors worth 150% of the cost of the equipment i.e., in the case of train rotary door operator. This is not commercially justified and the increase in cost of the final product would reduce the attraction for customers. A commercially viable FDI solution should be incorporated with minimum extra cost, e.g., using processing power and sensors already available. This severely reduces the practicality of installing sensitive sensors such as airflow or acoustic transducers in industrial applications within limited installation and maintenance costs. The best option is to employ an array of position sensors to identify either the time taken by the STME in travelling a certain distance within each region of interest in the transitional path or finding the average velocity between two position sensors. The residuals also need to be robust to the variations in their installation precision. The average velocity over larger spatial regions inherently provides such robustness to modelling errors and noise.

The number of independent outputs of the system governs the extent of available analytical redundancy[xii]. The examples of discrete position sensors that may be used to form an array include limit switches, Hall effect sensors, optical sensors, magnetic sensors etc. Their mechanical design can usually be tailored to fit in the transitional path of an STME in a variety of mechanical configurations.

Managing Degree of Freedom

Degree of freedom relates to our capability to observe or model the energy paths within the system. Higher the number of energy paths being modeled leaves to us higher degree of freedom, but increasing the time and modeling complexity rendering it useless for today's world's need for minimum time to market. Since the number of residuals that can be uniquely isolated are dependent on the number of independent outputs of the system, increasing the number of sensors required.

In case of fault excluding the sensor or actuator fault, the model has already perturbed many and thus changing the spectra of the system in first place. The system normally tends to be more and more non-linear in case of a fault.

Establishing the independence of observations

Now comes an important issue of managing the independence of residuals while observing a unique profile that can be represented very coarsely by even a first order model of the system.

The STME throw has non-linear dynamics that makes the modeling very complex and especially its relationship with the mechanical energy distributors such as damper (D), mass (M) and spring (K). The dynamics of throw profile are captured using limit sensors placed at the optimal positions along the transitional path. The placement of the sensors along the path is discussed in later part of this paper. However, it was found that by increasing the number of sensors along the path, the residuals become more and more dependent. In order to maintain orthogonality, the sensors are only placed at key locations that are usually subject to change in either case of fault, failure or malfunction.

As in case of classical modeling, all energy paths cannot be modeled and observed due to practical, commercial or implementation concerns, similarly in this case by increasing the number of sensors would not be able to capture the dynamics or generate residuals for some of the energy distribution paths in the system. As in the case of STME, if the position sensors are increased beyond a certain number (depending on the non-linearity of the throw trajectory), it results in mutually dependent residuals that are not useful for FDI.

The optimum number of sensors therefore depend primarily on the following factors: -

- Non-linearity of the throw trajectory;
- Mutual dependence of residuals;
- Symmetry of forward and reverse throw, asymmetric cases may have more or less sensors depending on the locations of the regions of interest in the forward and the reverse throw trajectories.

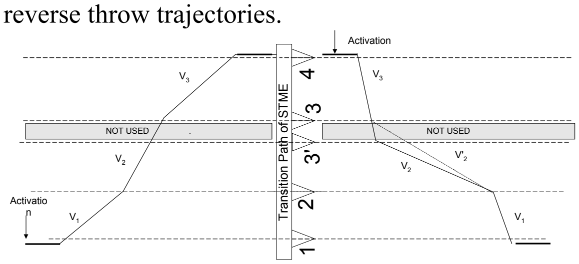


Figure 1. Residual generation using an Array of Position Sensors

Problem of Symmetry

Just as in the case of electric circuits, the charging and discharging cycle defines the time constant of the system. Therefore, the discharging is directly related to the resistance to discharge of energy and the capacity or memory to store energy. Complex energy storage mechanism leads to more and more non-linearity in the system.

In case of STME, the charging can be modeled as the linear dynamic process, but the reverse throw transition can vary due to release of stored energy or sudden force reversals normally achieved by different mechanical coupling or re-routing of energy. The reason why symmetry becomes important is evident. The figure shows a typical case of K-dominant equipment such as a train-stop. The regions of interest for the forward throw are significantly different from those of the reverse throw. The best option is to add a sensor to capture the reverse throw dynamics. The extra residual generated in forward and reverse throws may be discarded to eliminate unnecessary processing and data acquisition requirements.

Specifying Best Sensor Location

The best location can be achieved by three methods,

- By observing the throw trajectory to find the piece-wise linear regions approximately ;
- By finding the regions with close to zero acceleration bounded between two abrupt changes in acceleration (zero cross-over);
- By estimating the STME exponential constant (k) over the complete operating range and placing the sensors at the boundaries of abrupt changes in the estimates of k.

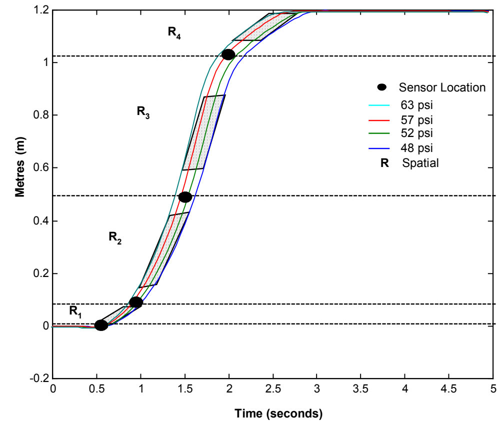


Figure 1a. Identification of Sensor Location using Heuristics and Visual Shape

It was found that these approaches gave almost identical results, yet with increased off-line processing and mathematics involved. The example of a train rotary door operator is explained here. It is obvious that the throw trajectory contains at least four significantly linear regions R_1, R_2, R_3, R_4. The position sensors can be placed at their boundaries. To identify the regions correctly, it is essential to use a model that can reasonably capture the system dynamics while maintaining sufficient independence required for the problem in hand. The aim of this intermediate system model is only to identify the regions for further algorithms, therefore the efficiency, the black-box nature of the model and the associated effects are irrelevant. Radial basis Functions (RBF) Neural Networks are therefore an obvious choice [xiii,xiv]. They are employed for modeling where the input represents the operating force u and the output is an m-dimensional vector representing the throw dynamics. The m samples of data are acquired at an interval Δt starting at time t_o. The spread of RBF model is selected such that the model is reasonably generalized over the complete operating region.

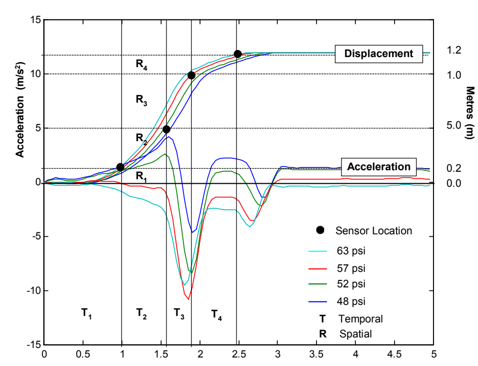
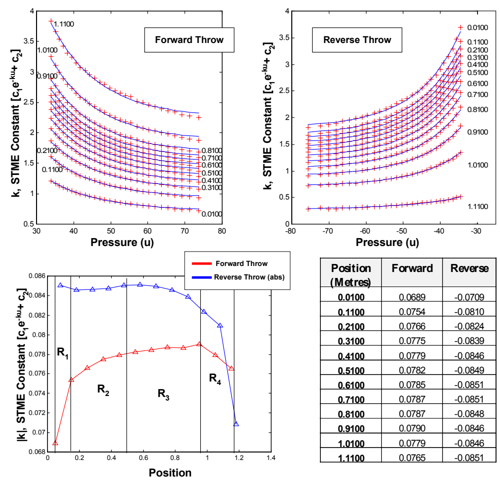


Figure 2. Identifying sensor location using acceleration and zero cross-over

A good test of the model is at the boundaries of the input-output map. Let n be the total zero-crossovers $y''_i(u)y''_{i+1}(u)<0$) then for each crossover k, where k=1,2,3,4,.. ,n, the temporal and spatial boundaries are extracted from the distance measurement . These boundaries are used as a guideline for placing limit switches at appropriate locations depending on physical constraints. Broadly speaking, the sensors must be deployed at the boundaries of the regions with almost zero acceleration.

The best method for identifying the sensor location numerically is to use the STME constant. A single sensor is gradually moved along the trajectory path. The STME constant evaluated with each sensor position is plotted against distance. This curve gives the best estimate of sensor location with maximum independent residuals.

Displacement	Spatial (inches)	0.02	0.12	0.5	1	1.180



Position (Metres)	Forward	Reverse
0.0100	0.0689	-0.0709
0.1100	0.0754	-0.0810
0.2100	0.0766	-0.0824
0.3100	0.0775	-0.0839
0.4100	0.0779	-0.0846
0.5100	0.0782	-0.0849
0.6100	0.0785	-0.0851
0.7100	0.0787	-0.0851
0.8100	0.0787	-0.0848
0.9100	0.0790	-0.0846
1.0100	0.0779	-0.0846
1.1100	0.0765	-0.0851

Figure 3. Best Sensor Location using STME Constant, k

The evaluation of sensor locations using this technique is presented here. The top two figures are a simulation of the sensor being moved along the trajectory path with a uniform spacing of 0.1 meters. The STME constant at each position is evaluated using the constrained optimization technique. The objective is to find a set of design parameters x_1, x_2, x_3. A general problem for STME characteristic property identification ($k=x_2$) is to minimize the cost-function,

$$J = \sum_{k=1}^{n} [(x_1 e^{-x_2 u_k} + x_3) - t_{r_k}]^2 \qquad (7)$$

where k=1,2,3,.. n and u_k and t_{rk} are the k^{th} force and its associated throw time. The only constraint is based on the general behaviour of the STME,

$$G_i x \le (x_3 - t_{r\min}) \qquad (8)$$

The absolute value of the STME constant k for forward and reverse throw is plotted against the distance. The resulting curve gives a reasonable measure for placing an array of sensors giving reasonably independent residuals. This is also a measure of the variance of residuals; the crisp curve reflects lower variance of the residuals and vice versa.

Parameter Estimation Approach for STME

Parameter Estimation [xv] can play a significant role in identifying the system faults as well as life cycle stage of STME. The model varies with a large number of operations. Although the model deviation over few thousand operations may be negligible, but over its life cycle, they show a reasonably measurable change[xvi]. This can help in finding its life-cycle stage. The true model parameters of STME can be given as M^T,

$$M^T = M + \Delta M \qquad (9)$$

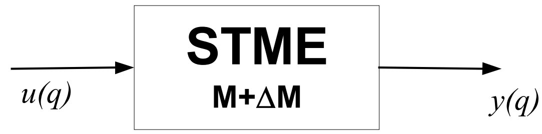


Figure 4. Static Model Identification

To find *ΔM,* we can use the primary residual and assume that $y^T(q)$ is the output from true plant for q^{th} operation,

$$o_m(q) = y^T(q) - Mu(q) = Mu(q) + \Delta Mu(q) - Mu(q) = \Delta Mu(q) \qquad (10)$$

Therefore, for change estimation from parity-space residuals using least squares is,

$$\Delta M = [\Phi'\Phi]^{-1}\Phi' O_m \qquad (11)$$

Where,

$$O_m = O_m(q,N) = \begin{bmatrix} o_m(q) \\ \mathrm{M} \\ o_m(q-N+1) \end{bmatrix} \qquad (12)$$

$$\Phi = \Phi(q,N) = \begin{bmatrix} u'(q) \\ \mathrm{M} \\ u'(q-N+1) \end{bmatrix}$$

The true model parameters M^T,

$$M^T = M + \Delta M \qquad (13)$$

To find *ΔM* we can use the primary residual equation and assume that $y^T(t)$ is the output from true plant,

$$o_m(t) = y^T(t) - Mu'(t) = Mu'(t) + \Delta Mu'(t) - Mu'(t) = \Delta Mu'(t) \qquad (14)$$

Therefore for change estimation from parity-space residuals using least squares is,

$$\boxed{\Delta M = [\Phi'\Phi]^{-1}\Phi' O_m} \qquad (15)$$

Where,

$$O_m = O_m(t,N) = \begin{bmatrix} o_m(t) \\ \mathrm{M} \\ o_m(t-N+1) \end{bmatrix} \qquad (16)$$

The inverse function is usually non-linear and, therefore, requires an approximation,

$$\Delta M = Q\Delta\theta \qquad (17)$$

where $Q = \left.\frac{\partial M}{\partial \theta}\right|_{\Delta\theta=0}$

thus

$$o_\theta(t) = \Delta Mu'(t) = u'(t)Q\Delta\theta \qquad (18)$$

Therefore estimate of Δθ using least squares is,

$$\boxed{\Delta\theta = [Q'\Phi'\Phi Q]^{-1} Q'\Phi' O_\theta} \qquad (19)$$

The equations clearly show the implementation of change in process parameters and underlying parameters using parity space equations.

Linear STME Model

The STME used here for data acquisition and modelling is a Train Rotary Door Operator (TRDO) modelled for operating range of 30-75 psi. The STME characteristic curves allow the visualisation of STME dynamics in a simple 2D presentation for forward and reverse throws based on its model structure.

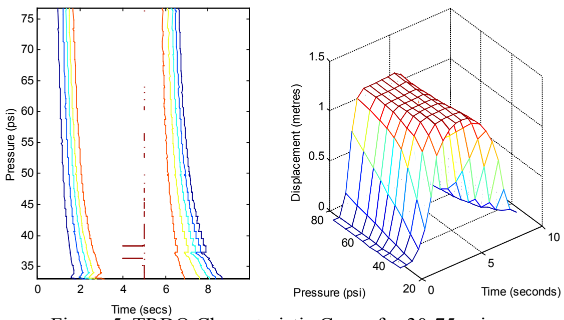


Figure 5. TRDO Characteristic Curve for 30-75 psi

The velocity measurements in various regions are fitted with a first order polynomial. The eigen values of the correlation matrix reflects the strong dependence of the residuals for all regions, but in case of a fault, they exhibit strong independence and principal component of each sensor can be related to a certain distinct module or physical entity. The parity space relation based on system inputs and outputs can be expressed as,

$$o_1 = y_1 - m_1 u \qquad (20)$$
$$o_2 = y_2 - m_2 u$$

If Δu is the input sensor fault and Δy is the output sensor fault in the system, then,

$$\begin{aligned} o_1 &= \Delta y_1 + m_1 \Delta u \\ o_2 &= \Delta y_2 + m_2 \Delta u \\ &\vdots \\ o_n &= \Delta y_n + m_n \Delta u \end{aligned} \qquad (21)$$

To achieve an appropriate structure and make certain residuals sensitive or insensitive to certain faults, *enhancement* is required. This is achieved by elimination of variables. To generate a residual not sensitive to input sensor fault,

$$\begin{aligned} \frac{o_1}{m_1} &= \frac{\Delta y_1}{m_1} + \Delta u \\ \frac{o_2}{m_2} &= \frac{\Delta y_2}{m_2} + \Delta u \\ r_{n+1} &= \frac{o_1}{m_1} - \frac{o_2}{m_2} = \frac{\Delta y_1}{m_1} - \frac{\Delta y_2}{m_2} \end{aligned} \qquad (22)$$

The slopes of each region around a nominal operating point are also the Eigen values if all sensors are considered independent and not coupled. But usually the extent of coupling can be extracted from correlation coefficient and keeping the mechanical model of the STME as black box. The parameters estimated that in a diagonal form, also equivalent to the eigen values of the system are,

	v_1	V_2	v_3	v_4
	R1	*R2*	*R3*	*R4*
$\boldsymbol{m_n,\lambda_n}$	0.0041	.0081	0.012	0.0027

In parity space approach the important is the independence of the model to faults rather than the system it self.

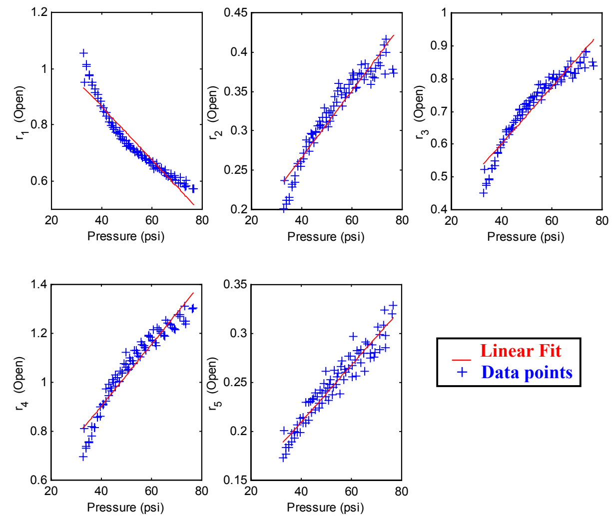


Figure 6. Forward Throw Model (TRDO Open)

Statistical Properties of Residuals

The eigen values acquired using parameter estimation for healthy systems are subject to their own variance limiting the threshold for faults and failures in first place.

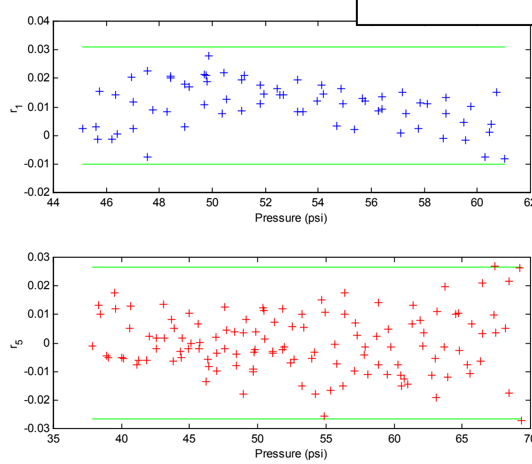


Figure 7. Residuals and the model structure left un-modeled

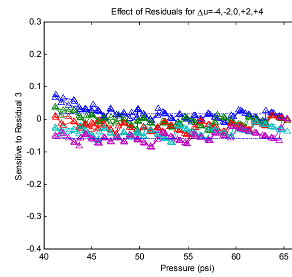
The statistical properties of the residuals are used to determine residual thresholds. The fixed residual thresholds for all case studies are evaluated for probability of type I error (false alarm) α=0.01, 1-α=.99 and corresponding z(D) [where D(z)=Φ(z)-Φ(-z)] is 2.576 (from the statistical tables). Therefore, in this case a general term for all residuals in all case studies is,

$$P\left(\mu_r - k \le \mu_r \le \mu_r + k\right)_{\mu_r} = \Phi\left(\frac{k}{\sqrt{\sigma^2/n}}\right) - \Phi\left(\frac{-k}{\sqrt{\sigma^2/n}}\right) = .99 \qquad (23)$$

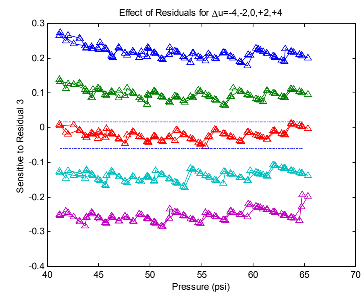
$$T_{U_r} = \mu_r + 2.576 * \sqrt{\sigma^2/n}$$

$$T_{L_r} = \mu_r - 2.576 * \sqrt{\sigma^2/n}$$

This shows that models are not good enough and can be improved either by reducing the operating region or increasing the order of model. Another option is to use exponential modelling approach for STME. Some STMEs such as TRDO usually have large operating regions; therefore improving model quality is very crucial.



Error Δu **EFFECTS** o_3



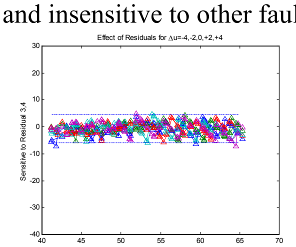
Residual is effected by Δy_3

Figure 7. Residuals and their sensitivity to numerous faults

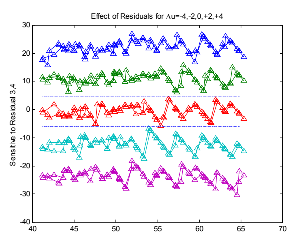
Fault Detection and Isolation (Linear Model)

		LIMIT SWITCHES					INPUT
		Actv Dly	.02→.12	.12→.5	.5→1	1→1.18	Pressure
	Δy	Δy_1	Δy_2	Δy_3	Δy_4	Δy_5	Δu_1
	-C	1.2459	0.0966	0.2565	0.3987	0.0936	
Independent Residuals	o_1	1	0	0	0	0	-0.0095
	o_2	0	1	0	0	0	0.0042
	o_3	0	0	1	0	0	0.0086
	o_4	0	0	0	1	0	0.0126
	o_5	0	0	0	0	1	0.0029
Dependent Residuals	r_5	0	0	116.28	-79.37	0	0
	r_6	0	238.10	-116.28	0	0	0
	r_7	0	238.10	0	-79.37	0	0

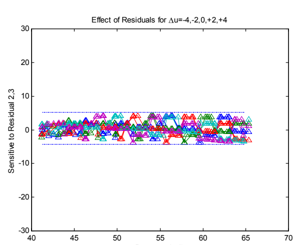
The first five independent residuals are used to generate other dependent residuals that can be made sensitive to certain faults and insensitive to other faults.



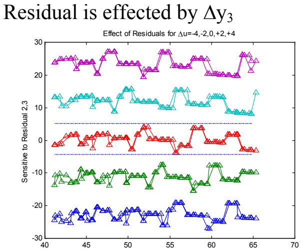
Error Δu had no effect on r_5



Residual is effected by Δy_3



Error Δu had effect on r_6

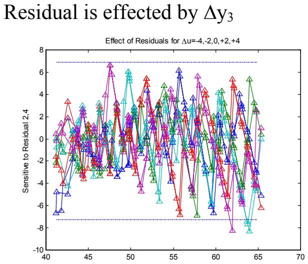


Residual is effected by Δy_3

KEY

. Δu=+4 __ Opening Operation
Δu=+2 __ Opening Operation
Δu=0 __ Closing Operation
Δu=-2 __ Closing Operation
Δu=-4 __ Closing Operation

__ Adaptive Threshold
__ Fixed Threshold



Residual is **NOT** effected by Δy_3

The residuals r_5-r_7 are made insensitive to pressure sensor failures. Two faults are induced in the system Δu and Δy_3. The effect of this fault Δy_3 is on residuals except residual r_7 that is insensitive to Δy_3.

Further work

The problem of STME can easily be extended to rotating machines. A rotating machine can be considered in steady state as simple harmonic motion and sensors placed could be used to generate temporal relationships and thus a set of parity space relations very similar to described here in this paper. Any change in rotational behavior would generate residuals that can be used to identify and isolate faults.

Conclusion

This paper attempts to model the STME as single input and multiple output system to ensure that enough non-zero eigen values are available in order to manipulate the system into desired structure. The system involves re-estimation periodically in order to identify the direction of rotation of eigen vectors under a fixed basis. It has however been found that the residual generator managed to yield sensitive threshold to numerous sensor and system errors and exhibited repeatable detection of faults under varying pressures.

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