Simulation uncertainty of complex economic system behavior

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1. Introduction

Property relation management of the (ownership, disposable, using) limited resources, which naturally occurs in the economic systems, face a problem uncertainty behavior of its active elements.

The model of identification and forecasting of the economic system trajectory states in time is offered in work, which allows complex estimation its conduct from positions of risk (additive distributing), incompleteness (subadditive distributing) and contradiction (superadditive distributing) of information accessible to the manager.

2. Uncertainty development of the economic systems: reasons and consequences

2.1. Information as the universal resource of the economic system

The key distinctive feature the economic system (*ES*) of any level (micro, middle, macro, mega levels) is question of ownership which arises up between active elements (people) in the process of production, distributing, redistribution, exchange and consumption of products and determines the behavior of the system. The ES resources have specific characteristics such as narrow-mindedness, liquidity, profitability, elasticity and inertia.

Time is the *most limited* resource. It is explained it foremost by narrow-mindedness of life time which figures in ES. *A money* and other financial instruments provide the most mobile access to the various limited resources are the *most liquid* resource. *A human capital* or knowledge is the *most cost-effective resource*, the transmitters of which are managers, experts, consultants. *Energy* is the *most elastic* resource, for different types of which an empiric rule is executed that insignificant reduction of suggestion results in the considerable price advance. *Ecology* the changes of which accumulate gradually is an inertia resource. In a modern world, when an economic environment virtual substantially, all transferred resources ES are time, a money, labor, energy and ecology can be in wide enough limits transferable by information (knowledge) about the behavior of the system and influence on its environment. Information becomes the universal ES resource.

The deficit of ES behavior information considerably reduces management efficiency. There reason of such vagueness are objectively inherented ES difficulties : *inertia* is the considerable interval of time between influence and its consequences; non-linearity is the change of proportions between expenses and the ES results in time and space; a conflict is disparity and contradictions between the local aims of active elements (people) and global ES purpose; asymmetric of information distributing between the economic activity subjects (effects of commercial secret, aggressive advertising, PR and i.e.); the waving pattern of behavior is growth stages alternation, stabilization, slump in the ES development; the market environment chaos is unregulated and uncontrolled influence of external environment on ES.

Therefore it is impossible to fully clean up for an observer the vagueness of the ES behavior. It is possible only to reduce the level of vagueness. The most extreme limit of such decline is probabilistic distributing of the future ES states as "the most clear from unclear measures".

2.2. Reasons of uncertainty and limits of distinctness behavior of the economic systems

The uncertainty behavior of the economic system (ES) in consequence of well known in an economic theory contradiction between the unlimited necessities of large number of economic activity subjects and narrow-mindedness of materially-power and informatively-intellectual resources accessible to them (within the relation of ownership).

Uncertainty is fundamental description of material well-being of the management process by ES information or in more general interpretation is knowledge of its behaviour [Klir 1985]. Therefore it can be expressed as ***uncertainty*** (effect of "fog"), ***unauthenticity*** (effect of "mirage") and ***ambiguousness*** (effect of "fuzzy").

Uncertainty supposes complete or partial absence about ES behavior information, the reasons of which can be *incompleteness*, *unstructured*, *uninterpreted* and *immunity* of information. *Incompleteness* shows the level of presence of accessible for registration or necessary information about the ES behavior. *The unstructured* substantially hampers or practically eliminates access and present information using. *The uninterpreted* shows absence of univocal correspondence between the high-quality and quantitative constituents of informative aggregate, for example, between economic indicators and their values. *Immunity* is related to the form incomprehensible for an observer registrations and presentations of information, for example, measuring of value of index in units unknown to the user.

An unauthenticity supposes appearance of fictitious, i.e. not objectively reflecting the ES behavior, information. It can be the reasons of unauthenticity *inadequacy*, *contradiction*, *changeability* and *distorted* of information. *Inadequacy* brings to the loss of useful or appearance of fictitious information as a result of its transformations from one form into other. Inadequacy is selected: *measuring* (gomomoraphyzm of the empiric system in numerical is broken); *treatments* (the method of information treatment falls short of to the types of scales, which it is fixed in); *storages* (partial or complete unrestored of information after archiving or compression); *deliveries* (the form of conclusion of information falls short to the possibilities of its perception by an observer). *Contradiction* is determined as simultaneous truth and falsity of certain fact. Contradiction could be got directly (a fact and an antifact act on different communication channels, from different information generators) is *contradiction of the first famil*y, or it is mediated by a logical conclusion from the row of primary investigation facts which conflicts with other fact, is *contradiction of the second family*. For description behavior ES contradiction of the first family is related to symmetry of preference relation on the great number of the future states alternatives of the system or properties (attributes) of its purpose, and contradiction of the second family — nontransitive dependence preferences of development. *Changeability* describes the information senescence process in time, because of object analysis changeability. It is characterized by actuality and timeliness of information coming in about the ES behavior. An economic information distortion is unintentional and intentional. The last one is related to that economic indicators execute criterion function and for the subjects of management is a motivational factor of necessities satisfaction level.

An ambiguousness describes the objectively existent limits of uncertainty. A *stochastic* (ambiguousness of external environment) and *semiotic* (ambiguousness of sign description of external environment) ambiguousness it is selected. The reasons of stochastic ambiguousness are *a chance* (unorganized complication) and *inaccuracy* of measuring. A chance could be formalized as a probabilistic event in two ways. Within the framework of the classic chances theory based on asymptotic form by A.N.Colmogorov, probability appears as limit of appearance of some event frequency, and within the framework of subjective theory of chances (B.De Fynetty, G.Geffris, J.Kains) as personality estimation of expectations. For the analysis ES first approach is substantially limited by the hard and difficultly executable requirement of reproducing unlimited number of homogeneous events at saving unchanging complex of external conditions. A semeiotic ambiguousness is related to the sign information reflection about problem situation. The reasons of ambiguousness could be *complication of description, calculation* or information *interpretation*; discrete of information and *linguistic uncertainty* which is related to the natural language (NL) of observer.

So we can do some conclusion, that the variety of types of uncertainty the ES behavior supposes the variety of mechanisms of its formal description.

3. Model of behavior uncertainty of the economic system

3.1. Raising of problem

It is possible to represent ES of any kind of level formaly as "input-state-output" of structure with the feed-back. The conduct of such cybernetic structures is described in time by the operators of the H transitions and the G outputs (1):

$$\vec{z}(t_1)=H\left[\vec{z}(t_0),\vec{x}(t_0),\vec{y}^b(t_0)\right];$$
$$\vec{y}(t_1)=G\left[\vec{z}(t_0),\vec{x}(t_0),\vec{y}^b(t_0)\right], \tag{1}$$

where $\vec{z}(t)=(z_1(t),z_2(t),...,z_n(t))'$ — is a vector of ES state parameters in the moment of time of t; $\vec{y}(t)=(y_1(t),y_2(t),...,y_{m_1}(t))'$ — is a vector of ES outputs parameters in the moment of time of t; $\vec{x}(t)=(x_1(t),x_2(t),...,x_{m_3}(t))'$ — is a vector of ES inputs parameters in the moment of time of t; $\vec{y}^b(t)=(y_1{}^b(t),y_2{}^b(t),...,y_{m_2}{}^b(t))'$ — is a vector of parameters of the ES feed-backs in the moment of time t; t_0, t_1 — are the real and future moments of time.

Influence of external environment on ES is carried out through the entrances. Some part of this entrances belongs to the unobserved and uncontrollable classes, and other part which is interrelated with coming of financial, materially-energy sources, informatively-intellectual and labor resources, are partly observed and managed because of changeability and uncontrolled of resources markets.

The ES Influence on an external environment is carried out through outputs which it is possible to classify on: positive — products and services; neutral — pollution-free and negative — ecologically dirty wastes. The most important constituent here which determines the ES behavior are the feed-backs. Predominance of negative or positive feed-backs determines the periods of the evolutional or revolutionary ES development.

The reasons of uncertainty parameters of the H state and the G outputs of ES are related to: incompleteness of information about the resources markets (entrances) and markets of products, services and wastes (outputs); contradictory information about the current status ES because of subjectivism, insufficient culture level and ignorance of personnel; information connectedness because of the feed-back presence and its qualitative temporal changes.

Therefore the results of transition operator's application and output (1) for future ES states forecasting have the fuzzy distributing:

$$z_{i_1}(t) \Leftrightarrow \left\langle z_{i_1 j}(t), \mu_j(t), j = 1, n_{i_1} \right\rangle,\ \forall i_1 = \overline{1, n};$$

$$y_{i_2}(t) \Leftrightarrow \left\langle y_{i_2 j}(t), \mu_j(t), j = 1, n_{i_2} \right\rangle,\ \forall i_2 = \overline{1, m}, \tag{2}$$

where $z_{ij}(t)$, $y_{ij}(t)$ — j - possible value of the i - parameter of the ES output state; n_i — an amount of parameter possible values; $\mu_j(t)$ — fuzzy measure of appearance of j - parameter value.

It is necessary to get the fuzzy forecast of all substantial state parameters and the ES outputs.

3.2. Formal uncertainty estimation

For formal uncertainty ES behavior description we use conception of fuzzy measures [Sugeno 1977, Wang, Z. & Klir, G.J., 1992], which covers all known formal mechanisms of different types uncertainty description (measure of belief, plausibility, necessity, possibility, probability, fuzzy).

We will consider the universal set of events Ω from the ES knowledge base, which is named a sure event. The empty set $\varnothing$ equates with an impossible event. To every event $A \subseteq \Omega$ we will put in accordance the real number $\eta(A)$ being the fuzzy measure of level of authenticity A.

An fuzzy measure $\eta(A)$ names a function η: $A \to [0,1]$, which satisfies to the axioms of narrow-mindedness (3), monotony (4) and continuity (5):

$$\eta(0) = 0;\ \eta(\Omega) = 1; \tag{3}$$

$$\Omega \subseteq A \subseteq B \subseteq \varnothing \Rightarrow \eta(A) \le \eta(B); \tag{4}$$

$$\lim_{i \to +\infty} \eta(A_i) = \eta\left(\lim_{i \to +\infty} A_i\right), \tag{5}$$

where $\{A_i\}_n$ is sequence of the inlaid great numbers of kind $A_0 \subseteq A_1 \subseteq ... \subseteq A_n \subseteq ...$ or $A_0 \supseteq A_1 \supseteq ... \supseteq A_n \supseteq ...$.

Expression $\eta(A)$ represents the degree of event A fuzzy, which can be interpreted as judgment fuzzy estimation of "$A \in \Omega$" or degree of subjective compatibility of event A with a sure event Ω. Monotony (4) of fuzzy measure $\eta(A)$ draws implementation of the following inequalities system:

$$\eta(A \cup B) \ge \max\{\eta(A), \eta(B)\};$$

$$\eta(A \cap B) \le \min\{\eta(A), \eta(B)\},\ \forall\ A, B \in \Omega. \tag{6}$$

We will generalize the system (4) in case of arbitrary t - and s -norms:

$$\eta(A \cup B) \ge s\{\eta(A), \eta(B)\};$$

$$\eta(A \cap B) \le t\{\eta(A), \eta(B)\},\ \forall\ A, B \in \Omega. \tag{7}$$

Specification of the system (7) allows to get the different special cases of fuzzy measure $\eta(A)$. If in the system (7) the first inequality is strict, and the second grows into equality, we will get family of superadditive measures possessing property, the special cases of which are:

measures of belief [Shafer 1978] (lower probabilities [Dempster 1967]):

$$\mu(A \cup B) > \mu(A) + \mu(B) - \mu(A \cap B),$$

$$\mu(A \cap B) = \mu(A)\mu(B); \tag{8}$$

measures of necessity [Dubois and Prade 1988] (concerted functions of belief [Shafer 1978]):

$$\mu(A \cup B) > \max\{\mu(A), \mu(B)\},$$

$$\mu(A \cap B) = \min\{\mu(A), \mu(B)\}. \tag{9}$$

For superadditive measures $\{\mu(\bullet)\}$ with the values from $\{0,1\}$ are executed $\mu(A \cap \overline{A}) = 0$, $\mu(\mathrm{A}) + \mu(\overline{A}) \le 1$ and if $\forall A \in \Omega$. Therefore these measures allow adequately to estimate the future state ES in the conditions of incomplete great number of behavior system possible alternatives.

If in the system (5) the second inequality is strict, and the first grows into equality, we get family *of subadditive measures* $\{\nu(\bullet)\}$ possessing property $\nu(A \cup B) + \nu(A \cap B) < \nu(A) + \nu(B)$, the special cases of which are:

measures of plausibility [Shafer 1978] (upper probabilities [Dempster 1967]):

$$\nu(A \cup B) = \nu(A) + \nu(B) - \nu(A)\nu(B),$$

$$\nu(A \cap B) < \nu(A) + \nu(B) - \nu(A \cup B) = \nu(A)\nu(B); \tag{10}$$

possibility measures [Zadeh 1978, Dubois and Prade 1988]:

$$\nu(A \cup B) = \max\{\nu(A), \nu(B)\},$$

$$\nu(A \cap B) < \min\{\nu(A), \nu(B)\}. \tag{11}$$

For subadditive measures $\{\nu(\bullet)\}$ with the values from $\{0,1\}$ are executed $\nu(A \cup \overline{A}) = 1$, $\nu(\mathrm{A}) + \nu(\overline{A}) \ge 1$ for $\forall A \in \Omega$. Therefore these measures allow adequately to estimate the future state ES in the conditions of contradictory (surplus) great number of system behavior possible alternatives.

If the system of inequalities (7) assumes a system of equalities form we will get that determination: probability measures ρ for uninteractive events (additive measure); probability measures for interactive events ($A \cap B \ne \varnothing$) at confluence of the systems (8) and (10); classical degree of fuzzy belonging at confluence of the systems (9) and (11).

For the possibility measures $\nu(A)$, probability measures $\rho(A)$, necessity measures $\mu(A)$ at $\forall A \in \Omega$ the following including is executed:

$$\nu(A) \ge \rho(A) \ge \mu(A). \tag{12}$$

Probability describes the very narrow class of uncertainty, that explains narrow-mindedness of the mathematical methods of ES behavior forecast, based on mathematical statisticians.

In axiomatic determinations of fuzzy measures (8)–(11) different concrete prospects are present t - and s -norm (see table 2), that specifies on the adequate operators of their treatment.

3.3. Forecast of the ES state future

Forecast technology of the future ES state consists of the following stages:

(1) Selection of great parameters number of the state X and outputs Y ES.

(2) Determination of distributing values of future states parameters $\vec{z}(t)=(z_1(t), z_2(t),..., z_n(t))'$ and ES (2) outputs $\vec{y}(t)=(y_1(t), y_2(t),..., y_{m_1}(t))'$ by using fuzzy operators of the H transitions and the G outputs (1).

(3) Breaking up on the classes of great fuzzy numbers of the states X parameters and outputs Y ES in accordance with table 1.

Table 1 Classes of fuzzy of parameters and operators of their treatment

N	Classes fuzzy measures	Descriptions of uncertainty			Operators of treatment	
		plenitude	nonconflict	co-ordination	*t*-norms	*s*-norms
1	Clearness	1	1	1	$t = t_1$	$s = s_1$
2	Probability	1	1	0	$t = t_4$	$s = s_4$
3	Possibility	1	0	1	$t = t_4$	$s = s_5$
4	Plausibility	1	0	0	$t = t_3$	$s = s_4$
5	Necessity	0	1	1	$t = t_5$	$s = s_4$
6	Belief	0	1	0	$t = t_4$	$s = s_3$
7	Tie-up*	0	0	1	$t = t_2$	$s = s_2$
8	Complete fuzzy	0	0	0	—	—

* this class is entered by author for the first time starting from description plenitude of fuzzy measures and is required additional research.

Classes t -norm and s -norm are used for simulation is represented in table 2.

For any $\mu_1, \mu_2 > 0$ a next order takes place.

The offered accordance between the fuzzy measures classes and operators of treatment (table 1) is determinations of fuzzy measures (3)–(11) investigation of and rules of choice of the most informing operator from possible:

– choice of operator of t*-norms* for the measure of possibility of $t < t_5 \Rightarrow t \in \{t_i\}_4$, $t_1 \le t_2 \le t_3 \le t_4 \Rightarrow t = t_4$;

– choice of operator of t*-norms* for the measure of verisimilitude of $t < t_4 \Rightarrow t \in \{t_i\}_3$, $t_1 \le t_2 \le t_3 \Rightarrow t = t_3$;

– choice of operator of s*-norms* for the measure of necessity of $s > s_5 \Rightarrow s \in \{s_i\}_4$, $s_1 \ge s_2 \ge s_3 \ge s_4 \Rightarrow s = s_4$;

– choice of operator of s*-norms* for the measure of trust of $s > s_4 \Rightarrow s \in \{s_i\}_3$, $s_1 \ge s_2 \ge s_3 \Rightarrow s = s_3$.

Table 2 Base operations t -norm and s -norm

Classes	Properties	Operation t- norm & s- norm
Idempotent norms	$t(\mu,\mu)=\mu$ $s(\mu,\mu)=\mu$	minization $t_5(\mu_1,\mu_2)=\min\{\mu_1,\mu_2\}$ maximization $s_5(\mu_1,\mu_2)=\max\{\mu_1,\mu_2\}$
Strictly monotonous Archimedean operations	$t(\mu,\mu)<\mu$ $s(\mu,\mu)>\mu$	$t_\gamma(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{\gamma+(1-\gamma)(\mu_1+\mu_2-\mu_1\mu_2)}$ $s_\gamma(\mu_1,\mu_2)=\dfrac{\mu_1+\mu_2-(2-\gamma)\mu_1\mu_2}{1-(1-\gamma)\mu_1\mu_2}$ $\gamma=1$ probabilistic multiplication $t_4(\mu_1,\mu_2)=\mu_1\mu_2$ probabilistic sum $s_4(\mu_1,\mu_2)=\mu_1+\mu_2-\mu_1\mu_2$ $\gamma=2$ $t_3(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{2-(\mu_1+\mu_2-\mu_1\mu_2)}$ the Lorents operator $s_3(\mu_1,\mu_2)=\dfrac{\mu_1\mu_2}{1+\mu_1\mu_2}$
Nil-Idempotent norms	$t(\mu,\mu)<\mu$ $s(\mu,\mu)>\mu$ $s(\mu,1-\mu)=1$ $t(\mu,1-\mu)=0$	limited difference $t_2(\mu_1,\mu_2)=\max\{0,\mu_1+\mu_2-1\}$ limited sum $s_2(\mu_1,\mu_2)=\min\{1,\mu_1+\mu_2\}$ strongly limited difference $t_1(\mu_1,\mu_2)=\begin{cases}\mu_1, & \text{if} \quad \mu_2=1;\\ \mu_2, & \text{if} \quad \mu_1=1;\\ 0, & \text{if} \quad \mu_1,\mu_2\neq 1\end{cases}$ strongly limited sum $s_1(\mu_1,\mu_2)=\begin{cases}\mu_1, & \text{if} \quad \mu_2=0;\\ \mu_2, & \text{if} \quad \mu_1=0;\\ 1, & \text{if} \quad \mu_1,\mu_2\neq 0\end{cases}$

(4) Parameters forecast of the state and the ES outputs through composition of fuzzy operators:

$$\bar{z}_{i_1}(t)=\mathop{S_N}_{j=1}^{n_{i_1}} T_N\left[\mu_j(t),z_{i_1 j}(t)\right],\ \forall i_1=\overline{1,n};$$

$$\bar{y}_{i_2}(t)=\mathop{S_N}_{j=1}^{n_{i_2}} T_N\left[\mu_j(t),y_{i_2 j}(t)\right],\ \forall i_2=\overline{1,m}, \tag{13}$$

where $N=1,\ldots,8$ is index of class of uncertainty;

S_N, T_N are operators *of* s *-norms* and t *-norms* (table 2), N — class of uncertainty (table).

(5) Determination of an administrative levers with a state parameters purpose selection, which are most sensible to the system entrances.

The matrix of cross-elasticity is determined for this purpose:

$$\left\| E_{i_1 i_2} \right\|_{n,m}, \tag{14}$$

where $E_{i_1 i_2} = \frac{\partial \overline{y}_{i_2}(t)}{\partial \overline{z}_{i_1}(t)} \times \frac{\overline{z}_{i_1}(t)}{\overline{y}_{i_2}(t)}$ is a elasticity coefficient, which shows on how many percents changes i_2 — out parameter $y_{i_2}(t)$ at the change of i_1 — parameter of the ES state on 1%.

The forecast findings (13) allow to get ES stochastic estimation (14) of the elasticity as an arc-coefficients $\left\| \breve{E}_{i_1 i_2} \right\|_{n,m}$:

$$\breve{E}_{i_1 i_2} = \frac{\overline{y}_{i_2}(t_1) - y_{i_2}(t_0)}{\overline{z}_{i_1}(t_1) - z_{i_1}(t_0)} \times \frac{\overline{z}_{i_1}(t_1) + z_{i_1}(t_0)}{\overline{y}_{i_2}(t_1) + y_{i_2}(t_0)}, \ \forall i_1 = \overline{1,n}; i_2 = \overline{1,m}. \tag{16}$$

(6) Using the results we‘ve got for the ES behavior management. For every output influence $y \in Y$ are influencing groups of the ES state parameters, which are represented in table 3.

Table 3 Influencing Groups of the state ES parameters

Group of influencing	The ES Outputs	
	positive	negative
effective	$\breve{E}_{i_1 i_2} > 1$	$\breve{E}_{i_1 i_2} < -1$
positive	$\breve{E}_{i_1 i_2} > 0$	$\breve{E}_{i_1 i_2} < 0$
neutral	$\breve{E}_{i_1 i_2} \cong 0$	$\breve{E}_{i_1 i_2} \cong 0$
negative	$\breve{E}_{i_1 i_2} < 0$	$\breve{E}_{i_1 i_2} > 0$
crisis	$\breve{E}_{i_1 i_2} << -1$	$\breve{E}_{i_1 i_2} >> 1$

The selection of such groups allows the active ES elements (owners, managers, experts) to manage of the system behavior and it is grounded to use *effective* parameters for purpose achievement with minimum resources expenses and strict values control over *crisis* parameters the insignificant worsening of which can result in the fatal consequences.

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