On an irreducible theory of complex systems

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1 Introduction

Complex systems profoundly change human activities of the day and may be of strategic interest. As a result, it becomes increasingly important to have confidence in the theory of complex systems. Ultimately, this calls for clear explanations why the foundations of the theory are valid in the first place. The ideal situation would be to have an irreducible theory of complex systems not requiring a deeper explanatory base in principle. But the question arises: where could such a theory come from, when even the concept of spacetime is questioned as a fundamental entity.

As a possible answer it is suggested that the concept of integers may take responsibility in the search for an irreducible theory of complex systems [1]. It is shown that complex systems can be described in terms of self-organization processes of prime integer relations [1], [2]. Based on the integers and controlled by arithmetic only the self-organization processes can describe complex systems by information not requiring further explanations. This offers the possibility to develop an irreducible theory of complex systems. In this paper we present results to progress in this direction.

2 A System of Equations: Nonlocal Correlations and Statistical Information about Parts of a Complex System

To understand a complex system we begin to consider the dynamics of the elementary parts and focus on the correlations between the parts as certain quantities of the complex system remain invariant [1], [2].

Let I be an integer alphabet and $I_N = \{x = x_1 ... x_N, x_i \in I, i = 1, ..., N\}$ be the set of sequences of length $N \geq 2$. We consider N elementary parts $P_i, i = 1, ..., N$ with the state of an element P_i specified in its reference frame by a generalized coordinate $x_i \in I, i = 1, ..., N$ (for example, the position of the element P_i in space) and the state of the elements by a sequence $x = x_1 ... x_N \in I_N$. It is proved [1] that $C(x, x') \geq 1$ of the quantities of a complex system remain invariant, if and only if a system of $C(x, x')$ equations take place

$$(m+N)^{C(x,x')-1}\Delta x_1 + (m+N-1)^{C(x,x')-1}\Delta x_2 + ... + (m+1)^{C(x,x')-1}\Delta x_N = 0$$

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$$(m+N)^1\Delta x_1 + (m+N-1)^1\Delta x_2 + ... + (m+1)^1\Delta x_N = 0$$

$$(m+N)^0\Delta x_1 + (m+N-1)^0\Delta x_2 + ... + (m+1)^0\Delta x_N = 0 \qquad (1)$$

characterizing in view of an inequality

$$(m+N)^{C(x,x')}\Delta x_1 + (m+N-1)^{C(x,x')}\Delta x_2 + ... + (m+1)^{C(x,x')}\Delta x_N \neq 0,$$

the correlations between the parts of the complex system, where $\Delta x_i = x'_i - x_i,\ x' = x'_1 ... x'_N, x = x_1 ... x_N,\ x'_i, x_i \in I, i = 1, ..., N$ are the changes of the elements $P_i, i = 1, ..., N$ in their reference frames and m is an integer. The coefficients of the system of linear equations become the entries of the Vandermonde matrix, when the number of the equations is N. This fact is important in order to prove that $C(x, x') < N$ [1].

The equations (1) present a special type of correlations that do not have reference to the distances between the parts, local times and physical signals. The space and non-signaling aspects of the correlations are familiar from explanations of quantum correlations through entanglement [3]. The time aspect of the nonlocal correlations may suggest new items into the agenda.

The solutions of the equations (1) may define for the observable Δx_i of an element $P_i, i = 1, ..., N$ a set of different possible values. The equations with different solutions give no rules to predict the outcome of the measurement of Δx_i. Nevertheless, they can provide the statistical information about the observable Δx_i, as long as it is possible to find from the solutions the probabilities of the different outcomes.

3 Self-Organization Processes of Prime Integer Relations and their Geometrization

The equations (1) can be also viewed as identities. Their analysis reveals hierarchical structures of prime integer relations in the description of complex systems [1], [2] (Figure 1). In the context of the hierarchical structures it may be useful to investigate whether the Ward identities and their generalizations [4] could be presented in a more explicit form.

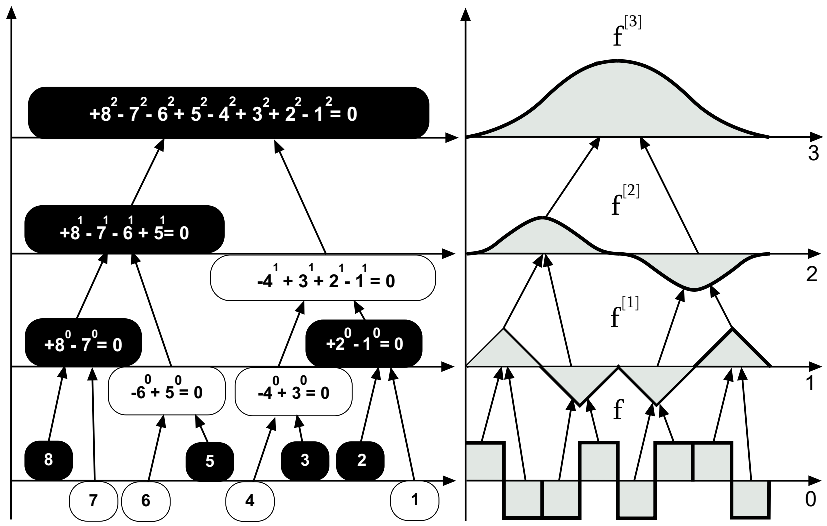

Figure 1: The left side shows one of the hierarchical structures of prime integer relations, when a complex system has $N = 8$ elements $P_i, i = 1, ..., 8,\ x = 00000000,\ x' = +1-1-1+1-1+1+1-1,\ m = 0$ and $C(x, x') = 3$. The hierarchical structure is built by a self-organization process of prime integer relations and determines a correlation structure of the complex system. The right side presents an isomorphic hierarchical structure of geometric patterns. On scale level 0 eight rectangles specify the dynamics of the elements $P_i, i = 1, ..., 8$. The boundary curves of the geometric patterns describe the dynamics of the corresponding parts. All geometric patterns are symmetric and their symmetries are interconnected. The symmetry of a geometric pattern is global and belongs to a corresponding part as a whole.

Through the hierarchical structures a new type of processes, i.e., the self-organization processes of prime integer relations, is revealed [1]. Starting with integers as the elementary building blocks and following a single principle, such a self-organization process makes up the prime integer relations of a level of a hierarchical structure from the prime integer relations of the lower level (Figure 1). A prime integer relation is made as an inseparable object: if even one of the prime integer relations is not included, then the rest of the prime integer relations can not form an integer relation. In other words, each and every prime integer relation involved in the formation of a prime integer relation is crucial.

By using the integer code series [5] the prime integer relations can be geometrized as two-dimensional patterns and the self-organization processes can be isomorphically expressed by certain transformations of the geometric patterns [1]. As it becomes possible to measure a prime integer relation by an isomorphic geometric pattern, quantities of the prime integer relation and a complex system it describes can be defined by quantities of the geometric pattern such as the area and the length of its boundary curve (Figure 1).

Due to the isomorphism, the structure and the dynamics of a complex system are combined. As self-organization processes of prime integer relations determine the correlation structure of a complex system, the transformations of corresponding geometric patterns may characterize its dynamics in a strong scale covariant form [1], [2].

4 Optimality Condition of Complex Systems and Optimal Quantum Algorithms

Despite different origin complex systems have much in common and are investigated to satisfy universal laws. Our description points out that the universal laws may originate not from forces in spacetime, but through arithmetic.

There are many notions of complexity introduced in the search to communicate the universal laws into theory and practice. The concept of structural complexity is defined to measure the complexity of a system in terms of self-organization processes of prime integer relations [1]. In particular, as self-organization processes of prime integer relations progress from a level to the higher level, the system becomes more complex, because its parts at the level are combined to make up more complex parts at the higher level. Therefore, the higher the level self-organization processes progress to, the greater is the structural complexity of a corresponding complex system.

Existing concepts of complexity do not explain in general how the performance of a complex system may depend on its complexity. To address the situation we conducted computational experiments to investigate whether the concept of structural complexity could make a difference [6].

A special optimization algorithm, as a complex system, was developed to minimize the average distance in the travelling salesman problem. Remarkably, for each problem the performance of the algorithm was concave. As a result, the algorithm and a problem were characterized by a single performance optimum. The analysis of the performance optimums for all problems tested revealed a relationship between the structural complexity of the algorithm and the structural complexity of the problem approximating it well enough by a linear function [6]. The results of the computational experiments suggest the possibility of a general optimality condition of complex systems:

A complex system demonstrates the optimal performance for a problem, when the structural complexity of the system is in a certain relationship with the structural complexity of the problem.

The optimality condition presents the structural complexity of a system as a key to its optimization. Indeed, according to the optimality condition the optimal result can be obtained as long as the structural complexity of the system is properly related with the structural complexity of the problem. From this perspective the optimization of a system should be primarily concerned with the control of the structural complexity of the system to match the structural complexity of the problem or environment.

The computational results also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Once the structural complexity could be controlled as a single entity, the optimization of a complex system would be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved in its description.

In the search to identify a mathematical structure underlying optimal quantum algorithms the majorization principle emerges as a necessary condition for efficiency in quantum computational processes [7]. We find a connection between the optimality condition and the majorization principle in quantum algorithms.

According to the majorization principle in an optimal quantum algorithm the probability distribution associated to the quantum state has to be step-by-step majorized until it is maximally ordered. This means that an optimal quantum algorithm works in such a way that the probability distribution p_{k+1} at step $k+1$ majorizes $p_k \prec p_{k+1}$ the probability distribution p_k at step k. There are special conditions in place for the probability distribution p_{k+1} to majorize the probability distribution p_k with intuitive meaning that the distribution p_k is more disordered than p_{k+1} [7].

In our description the algorithm revealing the optimality condition also uses a similar principle, but based on the structural complexity. The algorithm tries to work in such a way that the structural complexity $\mathbf{C}_{k+1}$ of the algorithm at step $k+1$ majorizes $\mathbf{C}_k \prec \mathbf{C}_{k+1}$ its structural complexity $\mathbf{C}_k$ at step k. The concavity of the algorithm's performance suggests efficient means to find optimal solutions [6].

5 Global Symmetry of Complex Systems and Gauge Forces

Our description presents a global symmetry of complex systems through the geometric patterns of prime integer relations and their transformations. It belongs to the complex system as a whole, but does not necessarily apply to its embedded parts. The differences between the behaviors of the parts may be interpreted through the existence of gauge forces acting in their reference frames. As arithmetic fully determines the breaking of the global symmetry, there is no further need to explain why the resulting gauge forces exist the way they do and not even slightly different.

Let us illustrate the results by a special self-organization process of prime integer relations [1], [2]. The left side of Figure 1 shows a hierarchical structure of prime integer relations built by the process. It determines a correlation structure of a complex system with states of $N = 8$ elements $P_i, i = 1, ..., 8$ given by sequences $x = 00000000$, $x' = +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1$ and $m = 0$. The sequence x' is the initial segment of length 8 of the Prouhet-Thue-Morse (PTM) sequence starting with $+1$. There is an ensemble of self-organization processes and thus correlation structures forming the correlation structure of the complex system. The self-organization process we consider is only one of them.

The right side of Figure 1 presents an isomorphic hierarchical structure of geometric patterns. The curvature of a geometric pattern determines the dynamics of a corresponding part of the complex system. Quantities of a geometric pattern, such as its area and the length of the boundary curve, define quantities of a corresponding part of the complex system. The quantities of the parts are interconnected through the transformations of the geometric patterns.

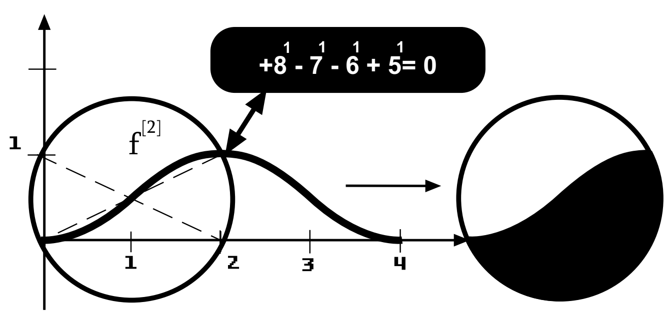

Figure 2: The geometric pattern of the part $(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$. From above the pattern is limited by the boundary curve, i.e., the graph of the second integral $f^{[2]}(t)$, $t_0 \leq t \leq t_4$ of the function f defined on scale level 0 (Figure 1), where $t_i = i\varepsilon$, $i = 1, ..., 4$, $\varepsilon = 1$, and it is restricted by the t axis from below. The geometric pattern is isomorphic to the prime integer relation $+8^1 - 7^1 - 6^1 + 5^1 = 0$ and determines the dynamics. If the part deviates from this dynamics even slightly, then some of the correlation links provided by the prime integer relation disappear and the part decays. The boundary curve has a special property ensuring that the area of the geometric pattern is given as the area of a triangle: $S = \frac{HL}{2}$, where H and L are the height and the length of the geometric pattern. In the figure $H = 1$ and $L = 4$, thus $S = 2$. The property is illustrated in yin-yang motifs.

We can see from Figure 1 that starting with the elements at scale level 0, the parts of the correlation structure are built scale level by scale level and thus a part of the complex system becomes a complex system itself. All geometric patterns characterizing the parts are symmetric and their symmetries are interconnected through the integrations of the function.

We consider whether the description of the dynamics of parts of a scale level is invariant as through the formation they become embedded in a part of the higher scale level.

At scale level 2 the second integral $f^{[2]}(t)$, $t_0 \leq t \leq t_4$, $t_i = i\varepsilon$, $i = 1, ..., 4$, $\varepsilon = 1$ characterizes the dynamics of the part $(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$.

This composite part is made up of elements P_1, P_2, P_3, P_4 and parts $P_1 \leftrightarrow P_2, P_3 \leftrightarrow P_4$ embedded in its correlation structure by the formations (Figures 1 and 2). The description of the dynamics of elements P_1, P_2, P_3, P_4 and parts $P_1 \leftrightarrow P_2, P_3 \leftrightarrow P_4$ within the part $(P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)$ is invariant relative to their reference frames. In particular, the dynamics of elements P_1 and P_2 in a reference frame of the element P_1 is specified by

$$f^{[2]}(t) = f^{[2]}_{P_1}(t_{P_1}) = \frac{t^2_{P_1}}{2}, \quad t_0 = t_{0,P_1} \leq t_{P_1} \leq t_{1,P_1} = t_1, \tag{2}$$

$$f^{[2]}(t) = f^{[2]}_{P_1}(t_{P_1}) = -\frac{t^2_{P_1}}{2} + 2t_{P_1} - 1, \quad t_1 = t_{1,P_1} \leq t_{P_1} \leq t_{2,P_1} = t_2.$$

The transition from the coordinate system of the element P_1 to a coordinate system of the element P_2 given by the transformation $t_{P_2} = -t_{P_1} - 2, \quad f^{[2]}_{P_2} = -f^{[2]}_{P_1} - 1$ shows that the characterization

$$f^{[2]}_{P_2}(t_{P_2}) = \frac{t^2_{P_2}}{2}, \quad t_{0,P_2} \leq t_{P_2} \leq t_{1,P_2} \tag{3}$$

of the dynamics of the element P_2 is invariant, if we compare (2) and (3). Similarly, the description is invariant, when we consider the dynamics of elements P_3 and P_4. Furthermore, it can be shown that descriptions of the dynamics of parts $P_1 \leftrightarrow P_2$ and $P_3 \leftrightarrow P_4$ relative to their coordinate systems are the same.

However, at scale level 3 the description of the dynamics is not invariant. In particular, the dynamics of elements P_1 and P_2 within the part $((P_1 \leftrightarrow P_2) \leftrightarrow (P_3 \leftrightarrow P_4)) \leftrightarrow ((P_5 \leftrightarrow P_6) \leftrightarrow (P_7 \leftrightarrow P_8))$ relative to a coordinate system of the element P_1 can be specified accordingly by (Figure 1)

$$f^{[3]}_{P_1}(t_{P_1}) = \frac{t^3_{P_1}}{3!}, \quad t_{0,P_1} \leq t \leq t_{1,P_1}, \tag{4}$$

$$f^{[3]}_{P_1}(t_{P_1}) = -\frac{t^3_{P_1}}{3!} + t^2_{P_1} - t_{P_1} + \frac{1}{3}, \; t_{1,P_1} \leq t_{P_1} \leq t_{2,P_1}.$$

The transitions from the coordinate systems of the element P_1 to the coordinate systems of the element P_2 do not preserve the form (4). For example, if under the transformation $t_{P_2} = t_{P_1} + 2, \; f^{[3]}_{P_2} = -f^{[3]}_{P_1} + 1$ the perspective is changed from the coordinate system of the element P_1 to a coordinate system of the element P_2, then it turns out that the description of the dynamics (4) is not invariant

$$f^{[2]}(t) = f^{[3]}_{P_2}(t_{P_2}) = \frac{t^3_{P_2}}{3!} - t_{P_2}, \quad t_{1,P_2} \leq t_{P_1} \leq t_{2,P_2}$$

due to the additional linear term $-t_{P_2}$.

Therefore, on scale level 3 arithmetic determines the different dynamics of the elements P_1 and P_2. Information about the difference could be obtained from observers positioned at the coordinate system of the element P_1 and the coordinate system of the element P_2 respectively. As one observer would report about the dynamics of the element P_1 and the other about the dynamics of the element P_2, we could find the difference and interpret it through the existence of a gauge force F acting on the element P_2 in its coordinate system to the effect of the linear term $\chi(F) = -t_{P_2}$

$$f^{[3]}_{P_2}(t_{P_2}) = \frac{t^3_{P_2}}{3!} - \chi(F), \ t_{0,P_2} \leq t_{P_2} \leq t_{1,P_2}.$$

In general, the results can be schematically expressed as follows:

Arithmetic $\rightarrow$
Prime integer relations in control
$\rightarrow$ *of correlation structures of complex systems* $\leftrightarrow$
$\leftrightarrow$ *Global symmetry:*
geometric patterns in control of the dynamics of complex systems $\rightarrow$
$\rightarrow$ *Not locally invariant descriptions*
of embedded parts of complex systems $\leftrightarrow$
$\leftrightarrow$ *Gauge forces to restore local symmetries*

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