Network Models of Mechanical Assemblies

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1.1. Introduction

Recent network research has sought to characterize complex systems with a number of statistical metrics, such as power law exponent (if any), clustering coefficient, community behavior, and degree correlation. Use of such metrics represents a choice of level of abstraction, a balance of generality and detailed accuracy. It has been noted that "social networks" consistently display clustering coefficients that are higher than those of random or generalized random networks, that they have small world properties such as short path lengths, and that they have positive degree correlations (assortative mixing). "Technological" or "non-social" networks display many of these characteristics except that they generally have negative degree correlations (disassortative mixing). [Newman 2003[i]] In this paper we examine network models of mechanical assemblies. Such systems are well understood functionally. We show that there is a cap on their average nodal degree and that they have negative degree correlations (disassortative mixing). We identify specific constraints arising from first principles, their structural patterns, and engineering practice that suggest why they have these properties. In addition, we note that their main "motif" is closed loops (as it is for electric and electronic circuits), a pattern that conventional network analysis does not detect but which is used by software intended to aid in the design of such systems.

1.1.1. Literature

Recent network analysis research has evolved from studying scale-free behavior [Albert and Barabási[ii]] to a search for understanding the functional behavior of systems. Various authors have studied software [Myers[iii]], the Internet [Li et al[iv]], biological systems [Milo et al[v]], [Ravasz et al[vi]], and electronic circuits [Ferrer I Cancho et al[vii]], among others. Each of these authors sought to find, with varying degrees of success, relationships between network statistics or structure, usually in the form of clusters (sometimes called modules) or internal patterns, and function. Deeper study has indicated that complex networks can be classified by domain, such as social, informational, biological, and technological, and their statistical properties compared [Newman 2003[i], Table II]. [Bourjault[viii]] introduced the "liaison" graph model of assemblies in which a part is represented by a node and a joint between two parts by an edge called a liaison, and used network and circuit theory to find all possible assembly sequences. [Björke[ix]] is one of many authors who use network methods to analyze tolerances and predict propagation of part size and shape variation in assemblies. Assemblies are equivalent to kinematic mechanisms, whose motion and constraint properties have been analyzed by network methods for decades [Phillips[x]], [Whitehead[xi]], [Blanding[xii]], [Konkar and Cutkosky[xiii]], [Shukla and Whitney[xiv]], [Whitney2004[xv]]..

1.2. Properties of Assemblies

1.2.1. Network Models

Mechanical assemblies are technological networks. Compared to the Internet, mechanical assembly networks are small, but they can be comparable in size to food webs and other recently studied networks, often having in the low to mid hundreds of nodes and edges.[1] Assemblies are typically designed to have a hierarchical structure, with subunits called subassemblies, which can be nested to a few levels (perhaps three to five, sometimes more). A hierarchical decomposition of an automobile might have top level assemblies {body, chassis, power train, interior} with respective main subassemblies {roof, car body side, car frame, doors, hood, trunk lid}, {springs, shock absorbers, steering gear}, {engine, transmission, drive axles, brakes}, and {seats, dashboard, steering wheel, shifter console, interior trim}, and so on.

A graph model of an assembly is usually formed by considering unitary parts[2] as nodes and joints[3] between parts as symmetric edges. Such models may be aggregated by collecting unitary parts into subassemblies, representing each subassembly as a collective node, and extending from this collective node only those edges that go to parts not in the subassembly. Graphs of assemblies are simple (no self-loops or multiple edges between nodes) and connected.

1.2.2. Statistical Properties

In this Section, we give in Table 1 the usual statistics about some assembly networks and compare them to Table 2 in [i]. The subject assemblies are, in increasing order of number of nodes, a rifle, a model airplane engine, an exercise walker, a

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[1] Assemblies are always modeled as undirected networks.

[2] A unitary part cannot be separated into two or more without using destructive methods that eliminate its identity and geometric coherence and thus end its ability to function as intended in the assembly.

[3] Joints typically exert kinematic constraint on the parts they connect, or exert enough mutual force to determine the joined parts' relative locations.

bicycle, and a V-8 automobile engine. These assemblies are modeled at the unitary part level and do not contain significant subassemblies. Exceptions are the coaster brake subassembly in the bike and items attached to the outside of the V-8 engine such as the air conditioning compressor and the alternator.

Assembly	n	m	Ratio of m to n and [$<k>$]	C*	Average path length	Random clustering coeff $<k>/n$	Degree correlation (Pearson)
1885 Winchester Single Shot Rifle	35	54	1.54 [3.09]	0.309	3.121	0.088	-0.177
Hyper 21 Model Airplane Engine	45	65	1.444 [2.889]	0.388	3.579	0.064	-0.235
Walker **	75	124	1.6 [3.2]	0.623	2.812	0.08	-0.213
Bicycle***	235	364	1.59 [3.175]	0.415	4.051	0.024	-0.202
V-8 Engine****	462	684	1.512 [3.024]	0.225	4.345	0.0124	-0.2695

Table 1. Statistical Properties of Some Assemblies. *Clustering Coefficient C calculated ignoring nodes whose nodal clustering coefficient = 0. **The walker assembly is symmetric and only half of it was analyzed. The version analyzed has 40 nodes and 64 edges. ***The bicycle was analyzed using only 10 identical spoke sets on each wheel, whereas the front has 32 and the rear has 40. Thus the version analyzed has 131 nodes and 208 edges. ***The engine network contains only 8 of the engine's 32 identical valve trains. Thus the version analyzed has 246 nodes and 372 edges. Many other small parts are also omitted. The above simplifications do not alter the conclusions. Clustering coefficient and path length calculated using UCINET v 6.9 [Borgatti et alxvi]. Pearson correlation calculated using software provided by Mark Newman.

1.2.3. Mean Degree of Assemblies

It was stated above that there is a limit on mean degree in assemblies. This constraint is discussed here, first empirically and then theoretically.

Figure 1 shows that the network connectivity (links/part) of assemblies having from 6 to 462 parts does not increase with the number of parts. In both real world scale free and random networks, it is predicted that connectivity should grow with the number of nodes. [ii] For the data available, mechanical assemblies do not behave this way. Assemblies with more than 100 or 200 parts are rare, for various engineering reasons. Figure 1 also shows visually the fact that, except for the Chinese Puzzle, network connectivities for these assemblies do not exceed ~2.1.

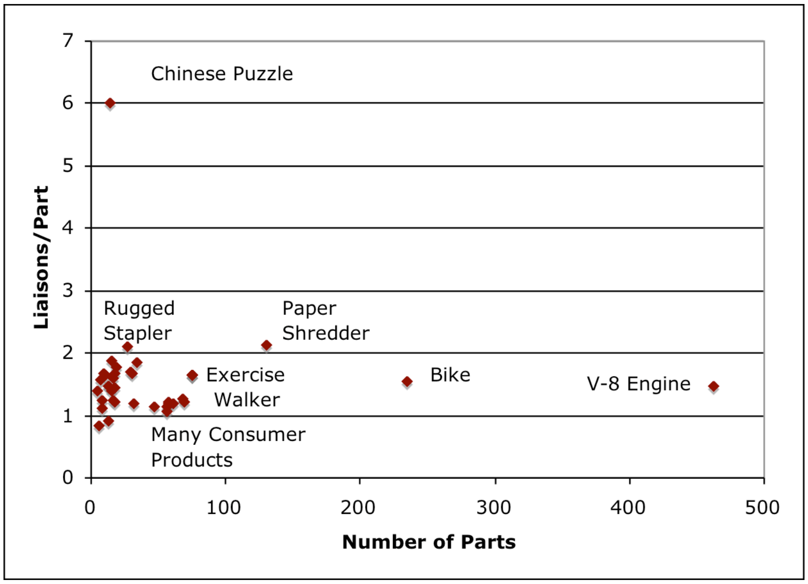

Figure 1. Liaisons Per Part vs Number of Parts for 35 Mechanical Products. There is no correlation between the network connectivity of these assemblies and the number of parts in them. The Chinese Puzzle is an outlier for reasons discussed in the text. The average connectivity for this dataset is 1.55, close to the value for the V-8 Engine (1.51). Data in this figure are a combination of those gathered by the author and those in [Van Wie et al. [xvii]]

Figure 1 shows that typical assemblies do not have anywhere near the connectivity that they might. To see a physical reason why, we begin by making the analogy between mechanical assemblies and kinematic mechanisms. All mechanisms are assemblies. While not all assemblies move in the sense that typical mechanisms do, assemblies nevertheless obey the same fundamental principles of statics. Among the issues of concern in kinematics is the state of constraint of the assembly: is it under-constrained and thus capable of movement; is it "exactly" constrained, having just enough links to prevent motion; or is it over-constrained, having more than enough links to prevent motion? The last case is considered undesirable [xi] [xii] because it could result in locked-in stress in the assembly, leading to assembly difficulties or field failure. The implication is that constraint plays the role of a limit or cost related to adding arcs to a node, as suggested by [Amaral, et al[xviii]] for other kinds of networks.
The Grübler criterion [x] is typically used to determine the numerical value of the number of degrees of freedom M in a planar mechanism:

Equation 1

$$M = 3(n - g - 1) + \sum \text{joint freedoms } f_i$$

where n = number of parts, g = number of joints, f_i = degrees of freedom of joint i

If $M > 0$, the mechanism has M under-constrained degrees of freedom. If $M < 0$, the mechanism has M more links than necessary to prevent motion and is over-constrained. If $M = 0$, the mechanism is exactly constrained. If we define α to be the number of joints (liaisons) divided by the number of parts (equivalent to the average network connectivity) and define the average number of degrees of freedom allowed per joint as β, then we may obtain from Eq (1)
Equation 2

$$g = \alpha n,\ \sum f_i = g\beta = \alpha\beta n, \text{ and}$$
$$M = 3(n - \alpha n - 1) + \alpha\beta n, \text{ or}$$
$$M = \alpha n(\beta - 3) + 3(n - 1)$$

Note that unless $\beta > 3$, increasing α will drive M negative and generate over-constraint. Furthermore, larger α and β mean more complex parts and a more complex product. If the mechanism is to be exactly constrained, then $M = 0$ and Equation can be solved for α to yield

$$\alpha = \frac{3 - 3n}{n(\beta - 3)} \rightarrow \frac{3}{3 - \beta} \text{ as } n \text{ gets large}$$

Equation 1

This expression is based on assuming that the mechanism is planar. If it is spatial, like all those in Table 1, then "3" is replaced by "6" and Eq (1) is called the Kutzbach criterion, but everything else stays the same. Table 2 evaluates Equation 1 for both planar and spatial mechanisms.

β	α **planar**	α **spatial**
0	1	1
1	1.5	1.2
2	3	1.5

Table 2. Relationship Between Number of Liaisons Per Part and Number of Joint Freedoms for Exactly Constrained Mechanisms (*M*=0).

Table 2 shows that α cannot be very large or else the mechanism will be over-constrained. If a planar mechanism has several two degree-of-freedom joints (pin-slot, for example) then a relatively large number of liaisons per part can be tolerated. But this is rare in typical assemblies. Otherwise, the numbers in this table confirm the data in Figure 1. Most assemblies are exactly constrained or have one operating degree of freedom. Thus $\beta = 0$ or $\beta = 1$, yielding small values for α, consistent with our data.

The Chinese Puzzle is an outlier because it is highly over-constrained according to the Kutzbach criterion. It is possible to assemble only because its joints are deliberately made loose. Nonetheless, the overabundance of constraints is the reason why it has only one assembly sequence, that is, why it is a puzzle.

2.1 Degree Correlation

Why do mechanical assemblies in Table 1 (and, most likely, many other) mechanical assemblies have negative r? Here we offer a number of promising suggestions. First, from a numerical standpoint, negative r goes with the tendency for highly connected (high $<k>$) nodes (sometimes called hubs) to link to weakly connected ones, and vice versa. This is true of all the assemblies studied so far. Many of the nodes attached to high degree nodes in an assembly are degree-one pendants, the presence of which tends to drive r negative. One contributing factor encouraging positive r is many tightly linked clusters. But such configurations are discouraged by the constraint conditions discussed in the previous section.

From a formal network point of view, assemblies typically are hierarchical and similar in structure to trees, although they rarely are pure trees. It is straightforward to show that a balanced binary tree has $r = -1/3$ asymptotically as the tree grows, and that a balanced binary tree with nearest neighbor cross-linking at each hierarchical level has $r = -1/5$ asymptotically. Such trees are similar to most distributive systems such as water, electricity, and blood.

From a functional/physical point of view, highly connected nodes in a mechanical assembly typically play either high load-carrying or load-distributing roles, or provide common alignment foundations for multiple parts, or both. The frames, pivot pins, and pedal arms of the walker, the frame and front fork of the bicycle, and the cylinder block, cylinder head, and crankshaft of the engine perform important load-carrying and alignment functions in their respective assemblies. Such highly connected parts are generally few in most assemblies, and they provide support for a much larger number of low degree parts. Almost without exception these highly connected nodes do not connect directly to each other. In high power assemblies like engines, there are always interface parts, such as bearings, shims, seals, and gaskets, between these parts to provide load-sharing, smoothing of surfaces, appropriate materials, prevention of leaks, or other services. Such interface parts are necessary and not gratuitous.

In addition, because they are often big, so as to be able to withstand large loads, the high-k parts have extensive surfaces and can act as foundations for other parts that would otherwise have no place to hang on. On the engine, such parts include pipes, hoses, wires, pumps and other accessories, and so on. Several of these must be located accurately on the block with respect to the crank but many do not. These are the degree-2 and degree-1 nodes that make up the majority in all the assemblies studied.

Summarizing, most assemblies have only a few high-k foundational, load bearing, or common locating parts, and many other parts mate to them while mating with low probability to each other. Thus even if the few high k parts mated to each other, the assortativity calculation still would be overwhelmed by many (high k – low k) pairs, yielding negative values for r.

Thus negative degree correlation in assemblies can be seen as a consequence of their tree-like structure, the presence of a few high-degree nodes and many low-degree nodes, and the absence of tight clusters, the latter caused by the need to avoid over-constraint.

3.1 Functional Motifs of Assemblies

In many technological networks, the motifs that generate function are closed loops. This is certainly true for both mechanical assemblies and electric/electronic circuits. The V-8 engine's main loops are shown in Figure 2. Some loops are contained entirely within the engine while others (air, output power) close only when other parts of the car or its environment are included. Note that some of them stay within communities identified by the Girvan-Newman algorithm [Girvan and Newman[xix]] while others extend beyond or link and coordinate different communities. These loops cannot be drawn by inspecting the graph but require domain knowledge.

The clustering coefficient of a network is obtained by counting triangles, that is, by enumerating the shortest possible loops. In general, the operative loops of an assembly are longer than three (typically 6 or 8) [xv] and thus do not contribute to the clustering coefficient. In fact, the conventionally defined clustering coefficient reveals nothing about the presence, number, or length of loops longer than three. Software for finding motifs[4] would be helpful here, but only some of the motifs thus found would be functionally significant, and domain knowledge would be needed to identify them.

Since positive degree correlation is related to more kinematic constraint while negative degree correlation is related to less kinematic constraint, the degree correlation calculation, when applied to mechanical assemblies, can be thought of as a simplified cousin of the Grübler-Kutzbach criterion, because the former simply counts links and considers them of equal strength, whereas the latter uses more information about both links and nodes and can draw a more nuanced conclusion.

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[4] http://www.weizmann.ac.il/mcb/UriAlon/groupNetworkMotifSW.html

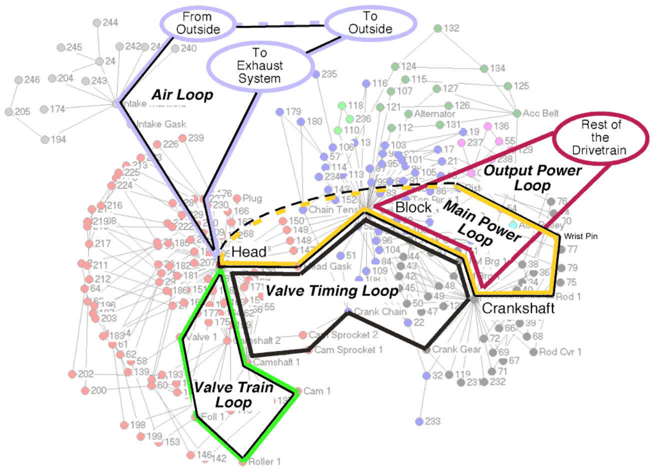

Figure 2. V-8 Engine with Five Main Functional Loops Indicated and Named. The main power loop shown is simplified, omitting parallel segments for the piston rings and an additional main bearing. It traces one of the 8 identical crank-piston-cylinder-head loops in the engine. The valve train loop is also greatly simplified, omitting parallel segments that include springs and slack adjusters. It traces one of the 8 identical valve trains in the network as illustrated here. The edge in the main power loop shown in dashes represents burning gas pressure and is not strictly speaking a mechanical link between two parts. But it is functionally indispensable.

4.1 Conclusions and Observations

Different kinds of systems have different operating motifs, and to understand a system is in some sense to know what those motifs are and how to find them. All of the assemblies analyzed here have a number of hubs. These are obviously important parts but they do not perform any functions by themselves. Instead, the identified functional loops are the main motifs, and they seem to include at least one hub, perhaps more. In systems where large amounts of power are involved, hubs often act as absorbers or distributors of that power or of static or dynamic mechanical loads. In other systems, the hubs can act as concentrators or distributors of material flow or information flow. Generally, material, mechanical loads, and power/energy/information all flow in closed loops in technological or energetic systems. All the assemblies analyzed display negative degree correlation. This follows from physical principles, the assembly's structure, or engineering design reasoning.

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