Analysing the behaviour of the textbook fractional reserve banking model as a complex dynamic system

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The Fractional Reserve Banking system has been used for several centuries by western banking systems to regulate the supply of loans against their deposits. It appears to be widely misunderstood, both within current economic theory and outside it. Modern economic textbooks base their explanation on a description that appears to have originated from a British parliamentary report authored in large part by John Maynard Keynes in 1931. This correctly describes how the system creates money through the re-deposit of loans made by banks, but predicts an evolution from initial conditions to a stable supply of money and credit for which there is no empirical evidence in the historical record. It notably fails to include the effects of either loan repayment and loan default, and consequently appears to incorrectly predict the long term behaviour of the process.

In this paper a simple debt flow model and accompanying computer simulation based on the textbook model is presented which includes loan repayment. Once loan repayment is included, stability issues can be seen as a result of flows of money between banks, and also from the operation of the repayment process itself. The model also demonstrates that the money multiplier for the system is a function of loan duration in addition to the the reserve requirement as described by Keynes, leading to different values than predicted by the standard model, and further sensitivities of the system's behaviour to both loan duration and the type of interest calculation being used.

1 Introduction

Fractional reserve banking has been used by European banking systems for over 400 years. It initially evolved out of practices used in gold based payment and storage facilities developed by medieval goldsmiths[8]. For the societies that first introduced it, it was a significant technological development that changed the monetary system from one that relied purely on physical precious metal monetary tokens, to one which allowed transfers of money to occur purely through book keeping, and paper based authorisations. Once fractional reserve banking was introduced, what had been a system that relied purely on physical transfers, became one that effectively had two components: physical money in the form of notes and coins, and book keeping entries representing bank deposits. Over time the proportion of physical money to money represented as deposits in bank accounts shifted until the present period, where for most currencies money is almost completely represented by electronic bank deposits and transfers.

The status of bank deposits as money, and the precise behaviour of the banking system in creating them, appears to be the cause of long standing confusion both within the field of Economics and outside of it. Fundamentally fractional reserve systems simply allow banks to create loans against money already held on deposit. The system developed out of the empirical experience of goldsmiths and early banks that it was possible to issue short term loans by taking advantage of the day to day reality that their customers would not all attempt to withdraw their money at the same time. In doing so, they essentially took advantage of a form of statistical multiplexing in providing access to deposits at the same time as lending a portion of them as loans, and as financial systems developed, and promissory notes were formalised, banknotes and bank deposits became to a large extent interchangeable. The early bankers also triggered a long running argument on the precise legality of this practice, and inadvertently unleashed a recursive feedback process onto the regulation of the money and loan supplies controlled by the banking system that has yet to be satisfactorily analysed.

Fractional reserve banking should not be regarded as just the simple redeposit of loans creating new deposits. The process creates a dynamic relationship between lending and bank deposits, and by extension the money supply whose behaviour is highly dependent on the regulatory framework surrounding it. Sucessful analysis requires a consideration of the larger system created by the flow of deposits and loans between banks and their regulation. A variety of systems of interbank lending and regulatory frameworks have emerged over the centuries as the result of repeated attempts to stabilise the amount of deposit expansion and consequent lending performed by the banking system. Several different regimes can be broadly distinguished, free banking which avoids the use of a central bank, 19th and early 20th century gold standard regulation, where gold was used to control physical currency issuance in conjunction with reserve requirements enforced by central banking regulations, First World War European emergency regimes which removed the direct tie within the system to the price and quantity of gold reserves, the post second world war Bretton Woods treaty which attempted to fix world currencies to both gold and the American dollar, and the current banking system which is based on the Basel series of treaties and generally relies on capital rather than reserve based regulation.

The formal recognition that the process of lending within a fractional reserve system caused an expansion in the amount of money represented on deposit as loans are re-deposited into the banking system is surprisingly recent. Although it was recognised that expansion was occurring in the late 19th century, the deposit expansion table typically presented today in introductory Economics textbooks[5] appears to have originated with the 1931 Macmillan report to the British Parliament[1] and was most probably authored by Keynes[10]. However, this expansion table only described deposit multiplication due to lending and its curtailment through reserve requirements. It did not include the effects of either loan repayments or loan defaults.

Incomplete as the Keynsian model may be, an alternative description that appears to have originated with Murray Rothbard[9] within the Austrian school of Economics, and which has circulated quite widely, is somewhat less useful by virtue of being factually incorrect. Rothbard wrote, as part of a general description of fractional reserve banking:

> Let's see how the fractional reserve process works, in the absence of a central bank. I set up a Rothbard Bank, and invest $1,000 of cash (whether gold or government paper does not matter here). Then I "lend out" $10,000 to someone, either for consumer spending or to invest in his business. How can I "lend out" far more than I have? Ahh, that's the magic of the "fraction" in the fractional reserve. I simply open up a checking account of $10,000 which I am happy to lend to Mr. Jones.

Rothbard's specific claim that in the absence of central banking individual banks are allowed to lend ten times the amount they have on deposit, rather than a fraction, has been widely circulated out of context as a general statement that individual banks can lend a multiple of their deposits. It can be disproved by an examination of any bank's accounts. It would also lead to exponential deposit expansion, and a consequent rapid collapse, if it was ever attempted in an actual banking system. He builds on this argument to then raise the issue that central bank intervention can also trigger the multiplier effect, Whilst this is one way that the deposit supply can be expanded, it and associated arguments in favour of the re-introduction of the gold standard, ignore considerable evidence that there are other issues with the stability and regulation of these systems, and that unstable behaviour was as much a feature of gold standard regulated systems as it is of current ones. Nevertheless, a large literature has arisen based on Rothbard's work within the Austrian school which has to be treated with some care with regard to the accuracy of the system they are describing. This is though, a general problem at present within Economics, where clear descriptions of the banking system under the different regulatory frameworks that have been used over the last two centuries are notable by their absence.

Confusion in this area even extends to official statistics of the monetary system, where there appears to be no standardisation of the various M series measurements being used across different currencies. Central bank statistics often use different components in different currencies for the same indicator, and in some cases, notably the USA's M2 measure, include components such as money market funds that represent short term debt. For the purposes of analysing the fractional reserve system, which creates an explicit relationship between the quantity of money on deposit at banks, and the amount of loans they can have outstanding at any given time, it is critical to maintain the distinction between the two. Money is a token of exchange, debt is a flow of money.[1]

Why however is the fractional reserve banking system in its various manifestations of particular interest as a complex system? In a market based monetary system, money is used as token of exchange - an intermediate form of information - to determine on a continuous basis a distributed solution to an N-Dimensional supply/demand equation across the economy. Money in this system is analgous to packets of information within a communication system, with the critical and necessary, restriction that the quantity of these tokens of exchange, money is fixed. If the quantity of monetary tokens being used to determine the price level is stable, then variations in supply and demand will be communicated as accurately as information latencies within the system permit. If the supply of goods for example generally increases, then prices will drop, and vice versa. If however, the total quantity of these tokens changes, it is impossible to determine purely from price information whether there has been a change in the quantity of money, the quantity of goods and services being exchanged, or both.

With the development of fractional reserve banking, and the ability to authorise direct transfers of deposits between bank accounts, the stability of the money supply became dependent on the stability of the banking system, and in particular the ability of its book keeping rules to manage the deposit/loan expansion process, the problem of loan default and consequent monetary contraction, and the consequent network flows of money created between banks as loans were repaid. In many respects, fractional reserve banking developed as a distributed system of monetary control, within a networked system of monetary flows and this makes it eminently suitable for consideration with the complex systems framework.

From a complex systems perspective then, we can define the banking system as a form of network, where loans create links between banking institutions mediating agreed flows of monetary tokens over time. Unlike communication networks, where there is no restriction on the total number of packets that can be sent beyond network capacity limits, in this network, price information

[1]It is necessary to exercise some care on this topic especially when comparing international monetary statistics. Also when reading the works of 19th and early 20th century economists since their definition of money was often restricted to physical notes and coins. Arguments in favour of treating bank deposits as money only began to appear in the late 19th century[3], and even today there is more debate than might be expected about what exactly constitutes money.

is obtained through the flow of a relatively fixed number of monetary tokens through the network in continuous time. However, the actualy quantity of these monetary tokens at any given time point is in part a function of the network paths (debt agreements), whilst the quantity and size of the network paths, is in part a function of the quantity of monetary tokens, with the coupling being performed by an essentially recursive relationship between monetary tokens and permitted loan capital. The result is a dynamic feedback system operating within the centre of the economy on the money and loan supply, and hence by extension on the price level, supply and demand for goods and services, and ultimately since money is used extensively within Economics as a measure, on Economic theory itself.

Studying the banking system as a complex system however, is easier said than done. The exact implementation details of the system are not well documented, particularly with respect to the effects of regulatory changes. The standard textbook presentation appears to be a simplified version of the early 20th century British banking system which used required reserves on deposits, in conjunction with gold standard regulation, but does not include loan repayment or default, or the role of bank capital in the system. It also implicitly assumes an absolute separation between deposits, reserves and loans, which is not always maintained within the banking system where financial instruments representing debt are sometimes treated as inter-changeable with money. It also assumes that ownership of loans stays within the regulated banking system.

The majority of modern banking systems are based on the three Basel treaties. These introduced a fundamental change to the system by replacing reserve regulation with capital regulation.[2] Capital regulation requires that Banks control their lending based on a combination of the amount they have on deposit, and a multiple of their regulatory capital, the precise value of which is determined by the type of loans that they have issued against their deposits.

So although the textbook description is necessarily the starting point for modelling fractional reserve based banking systems, it must be acknowledged that it is not representative, and indeed from the findings below, could not be implemented as described for any practical banking system. Besides the issue that the textbook model predicts a convergence to an asymptotically stable supply of money and loans which has never been observed in any long term time monetary series from fractional reserve based currency systems, the first debt default that occurred would cause the system to irreversibly contract.

It is the absence of any explicit mechanism to handle debt defaults and the consequent monetary contraction within the system, that is indicative that there must have always have been other mechanisms operating within banking systems on the total quantity of bank deposits besides the explicit multiplication effect of lending. All banking systems based on fractional reserve processes

[2]Reserve requirements are still used by some Basel based systems, notably China and Brazil, but they have generally been removed or much reduced. Within the American system reserves are only required for Net Transaction Accounts which are a small percentage of total bank deposits.

appear to exhibit a more or less continuous expansion in the quantity of money represented on deposit, a phenomena that modern economics terms "endogenous monetary expansion". This expansion appears in both gold standard and non-gold standard regulated systems, as statistics from 19th century Britain clearly show[7].

In the rest of this paper, we will explore the results from a simple computer simulation of the textbook representation of the banking system with loan repayment included. We will show that this model, simplistic though it is, reveals a number of apparent issues with the textbook representation, several of which are pertinent to actual banking systems, including instabilities related to loan flows, a modified value for the money multiplier which includes a dependency on the duration of loans being made, and a potential money creating race condition in interbank lending. Based on these findings we suggest that it is highly unlikely that the mechanics of actual banking systems behave in the stable way predicted by the textbook model, and that the actual behaviour of any given regulatory framework will be sensitive to multiple conditions within the system.

2 The Textbook Fractional Reserve System

In the standard textbook model of the banking system, money is deposited with banks which is then lent out to borrowers. Individual banks are only allowed to lend a fraction of their deposits, and are required to keep a regulated percentage in reserve. In this model, the definition of money is the sum of all deposits and reserves held by the banks. As loans are made, additional deposits are created as loan capital is redeposited within the banking system. To prevent unlimited monetary expansion, and also to ensure that banks have sufficient funds to meet day to day demands by customers for access to their funds, a reserve is kept based on the quantity of deposits held by each bank. The possibility of confusion of money with debt within the system - for example by allowing reserves to be held in financial instruments that represent loans (e.g. treasury certificates) which occurs in today's banking system, is not included in the model, which implicitly assumes that money and debt are kept completely separate.

The model is typically presented in the form of a series of deposits, loans and reserves made between a set of banks, with a specified reserve requirement, as shown in Table 1.

Table 1: Expansion of bank deposits with 10% Reserve requirement.

Bank	Deposit (Liability)	Loan (Asset)	Reserve
A	1000	900	100
B	900	810	90
C	810	729	81
D	729	656	72
E	656	590	66
F	590	531	59

As loans are created against each new deposit, the resulting deposit expansion is progressively throttled by the reserve requirement. The limit on total monetary expansion by the banking system is presented in conjunction with this model as the theory of the money multiplier (M), which is expressed as $M = 1/r$ where r is the reserve requirement or ratio expressed as a fraction.[3] For example, where the reserve requirement is 10% or 1/10 the money multiplier is $M = 1/r = 10$

The formula is derived from the following expansion series, where x is the initial deposit into the system, and r is the fractional reserve requirement :

$$x + x(r) + (x(r))r + ... = x\sum_{k=0}^{\infty}(r)^k \tag{1}$$

which converges to

$$x/(1 - r) \tag{2}$$

By extension, and assuming a complete separation of money and debt within the model banking system, the model also predicts limits on the total quantity of loan capital that can be issued against the total amount of money held in deposits by the banking system as shown in Figure 1[6].

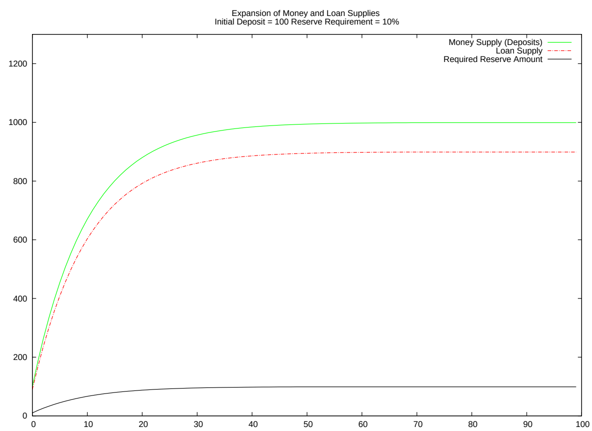

Figure 1: Theoretical limits of total bank deposits and loan supplies.

In particular, the total amount of loan capital originating within the banking system will always be a fraction of the total amount of deposits, provided that all loans issued by banks remain under the control of this regulatory framework. This relationship will be broken for systems that allow bank loans to be moved outside of bank regulation, for example by sale to non-bank entites. In modern

[3]Basel treaty regulation used in modern systems has effectively removed the reserve requirement as a regulatory factor, replacing it with capital regulation. Consequently many modern banking systems no longer have full reserve requirements.

banking systems, two mechanisms exist that may allow this, Asset Backed and Mortgage Backed securities, and in continental Europe, covered loans.

The final prediction made by the Keynsian model is that the system will expand to an asymptotically stable level of deposits and loan capital. This is perhaps the most interesting problem with the model, in that a noteworthy feature of all long term money supply time series is more or less continuous growth, punctuated by occasional sharp contractions.

3 The Textbook System with loan repayment.

In order to explore the behaviour of the textbook banking system when loan repayment is included, a mechanism must be created for money to be able to flow through the system. In the model presented here, Banks manage a number of deposit accounts held by employees, and can create loans to these employees subject to their regulatory requirements. Reserve percentages, loan interest rates, and loan duration can be set for each run. To create a flow of money through the system, on each round, assuming it has received sufficient interest payments from the previous round of loan repayments the bank makes salary payments to all loan holders which is sufficient for them to meet interest and capital repayments for this round for their outstanding loans. In effect, the interest payments received on loans extended by each bank in conjunction with the loan capital in the system is used in the next round to make payments to account holders so they can meet loan repayment requirements.

In this model Banks only make salary payments to their own account holders. Under different configurations of the model, lending can be made to account holders at other banks, or restricted to only be made to the Bank's own account holders - thereby effectively creating a closed system. Optionally, interbank lending can be enabled, allowing short term regulatory problems to be resolved by borrowing from other banks when the bank's deposit:loan ratio does not satisfy the reserve requirement. Employees may only have one loan at a time, and in each round banks lend to the maximum allowed by borrower availability and their regulatory requirements. The interest on all loans is calculated using the simple interest formula, $I = P * R * T$ where P is the principal of the loan, R is the interest rate, and T is the duration. Simple interest was chosen as it made checking the output of the model easier.

Applying loan repayments to the textbook model reveals several issues. To begin with, as shown in Table 2 there is an order of evaluation problem. If a bank first makes new loans, and then processes the repayment on its existing lending as shown on the left hand side of the table, it will immediately fall out of regulatory compliance when the loan repayment is processed. Conversely, if it makes the repayment and then processes the new loan, it will stay in regulatory compliance, but the rate of monetary expansion will be less than predicted by the non-loan repayment model, as the quantity of new loans will be based on a reduced deposit base.

Table 2: Order of Evaluation problem with loan repayment.

	Lend then Repay		Repay then Lend	
Bank	Deposit	Loan	Deposit	Loan
A	1000	800	1000	800
B	800*	810	800	700
C	810		700	

*Out of regulatory compliance with 10% reserve ratio.

Even if the more conservative approach is taken, and loans are made after repayments are processed, regulatory compliance cannot be maintained for very long. The textbook model is somewhat artificial, in that for clarity of presentation it presents a loan cascade where each bank makes a single loan to a depositor at the next bank in the series. However, as a consequence when loan repayments are applied, and assuming all banks are lending at the same rate of interest and loan duration, each bank except for Bank A, is in the position of receiving capital interest payments on a smaller loan, than its' depositor is making repayments on. Banks will consequently fall out of regulatory compliance with respect to the ratio of their deposits to their loans after only a few rounds of loan repayments.

A similar issue could be expected to occur in actual banking systems with any set of banks which allowed loans to be made to customers at other banking institutions. For example, if a single bank exists in a geographically separate area, it will be stable as long as all of its loans are made to its own customers. If a new bank opens up with access to its customer base, and either bank makes a loan to a customer at the other institution, then instabilities can be expected to arise at some point in the future purely as a result of unbalanced monetary flows between the two institutions as their customers repay their loans. In comparison, a Bank with an identical loan book, but whose loans were made only to its own customers would not experience these issues.

Daily imbalances in deposit/loan regulatory compliance are inevitable in all banking systems comprising multiple independent banking institutions, and are normally resolved by resort to the overnight interbank lending market. This facility in conjunction with the role of a central bank as a lender of last resort is usually regarded as a necessary part of the larger banking system.

There also appears to be a tendency for money to become concentrated at the bank which originates the largest loan, due again to the consequent flows of interest and capital repayments between banks. This parallels the historical development of the banking system where in many countries this bank subsequently became the central bank and took on a role as the lender of last resort to the rest of the banking system.

3.1 Examining the predicted value of the money multiplier

In order to isolate the behaviour of money multiplication from network flow effects a restriction was introduced to the model whereby banks only issued loans to their own deposit holders. This allowed the simplest form of the money and credit repayment behaviour to be studied, without the additional complication of debt flows.

With this restriction the behaviour of the money and loan supplies as the system evolved over time could be examined. Two clear patterns emerged, as shown in Figures 2 and 3. Figure 2 shows a run of the simulation with a loan duration of 12 accounting periods, an interest rate of 10% per annum, and 12 depositors. The initial deposits in the system are 10,000 monetary units, 5,000 of which are held by a single depositor and 5,000 by the bank. There are no constraints on bank lending beyond regulatory compliance, and the money supply expands to the maximum possible with this duration varying between 4.2 and 4.8 on successive loan repayment rounds.

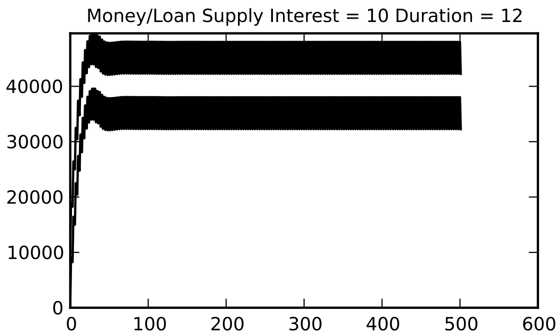

Figure 2: Monetary Expansion with 12 Depositors

In contrast, Figure 3 shows a run with the same parameters, except that there are only 11 depositors. Consequently since loan duration is 12 accounting periods, the bank is unable to make a loan in the 12th period, as no depositor can borrow. (Depositors may only have one loan at a time.) This triggers a cyclic contraction and expansion in the money supply with the money multiplier for the system varying between 3.3 and 4.9 as loans are repaid. Interestingly, this was not due to any form of loan default occurring, but occurred purely because there were no qualified borrowers for the bank to lend to.

In both cases, the money multiplier is less than predicted by the Keynsian model. This is due to the duration of loans being made, which is not sufficient to allow full expansion to occur. This potentially signicant as loan durations can vary significantly between banking systems, and also between different historical periods.

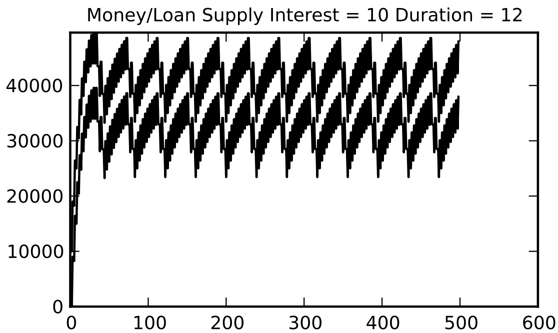

Figure 3: Monetary Expansion with 11 Depositors

3.2 Evolution of the Money Multiplier

Within the model, capital repayments are treated as a deduction in the loan amount outstanding and a matching deduction in the deposit amount held by the debtor. Interest payments effectively represent a movement of money between accounts, and so have no effect on the money supply in a simple interest model. With longer loan duration periods, the ability of the system to expand to its limits were explored, and this showed that the money multiplier was not only a function of the loan duration, but was also able to exceed the predicted theoretical limit of the standard model of $1/ReserveRatio$.

In the standard model which does not include any form of capital repayment, each new loan is made as the difference between the total amount of deposits, minus the reserve requirement, and the amount currently on loan. When loan repayments are introduced to the system, each accounting period causes the loan capital repayment amount to be deducted from both the total loan supply and the total money supply. However, as the money supply is always greater than the loan supply, the percentage change in the amount on loan is slightly lower for the money supply than it is for the loan supply. Consequently, when the next loan is made, based on the difference between the money supply and the loan supply, it is for a slightly larger amount than would have occurred without loan repayments.

For example, as shown in Table 1 with a reserve requirement of 10%, and an initial deposit into the system of 1000, the second loan made without capital repayments is 810. If before the second loan is made, a capital repayment of 100 is made on the first loan, then the total money and loan supplies will be 1800 and 800 respectively. The next loan amount is then $1800 * 0.9 - 800 = 820$, resulting over time and assuming a sufficiently large loan duration in a larger monetary expansion than occurs without capital repayment.

As shown in Figure 4, which is a run with 500 depositors, and a loan duration of 240 accounting periods, since the reserve requirement also acts as a constraint on lending, once the full monetary expansion of the system has been reached, a cyclic pattern is once again seen in the evolution of the money and loan supplies over time. (Since there are more depositors than loan accounting periods, this run cannot experience a shortage of borrowers.)

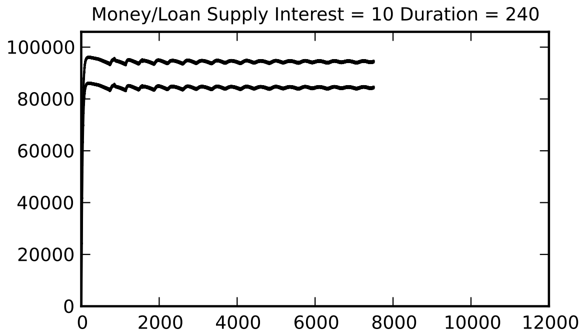

Figure 4: Behaviour with long duration (20 year) loan periods.

A further implication of this finding is that the type of interest calculation would also affect the model's behaviour - since loan repayments using compound interest calculations vary the quantity of the capital repayment component over the course of the loan.

4 Interbank Lending

Interbank lending is not normally treated as part of the standard textbook model, although as the example in Table 2 demonstrates, it is an integral part of the fractional reserve system when separate banking institutions exist within the same currency.

The status of a loan made by one bank to another lending institution however, within the fractional reserve model, is not the same as a bank loan to a normal depositor. Consider the following sequence of events.

Table 3: Interbank Lending Race Condition - Initial Conditions

Bank	Deposit	Loans
A	1014	750
B	742	675
C	669	

Bank B receives an interbank loan from Bank A for 8 units to cover the deposit/loan imbalance. It then receives an 8 unit interest repayment from Bank C. It can then repay the loan to Bank A which would restore the money supply to the previous amount. Or it can make a loan for 8 units to one of its own customers. This will causes its deposits to increase by 8 units, and it can then repay the loan to Bank A and still remain in regulatory compliance.

Table 4: Interbank Lending Race Condition - Loan from Bank A to Bank B

Bank	Deposit	Loans
A	1014	758
B	750	675
C	661	

It appears then that the earlier order of evaluation problem can also potentially lead to deposit creating race conditions in the interbank lending mechanisms, which can lead to deposit creation within the system independent of central bank control or the loan/depositor money creation mechanism.

5 Conclusion

The first central banks operating within the framework of a fractional reserve based system were the Sveriges Riksbank, established in 1668, and the Bank of England established in 1694. In the 400 years since then the system has been instrumental in the development of industrial society, providing mechanisms for the transfer of good ands services within the economy, and through savings mechanisms based on financial debt instruments, temporal transfers of goods and services. It clearly plays a central role within economic society, and this is at no time more evident than when the system, as it appears to do periodically, becomes unstable. Although the behaviour of the system was the subject of considerable research during the late 19th and early 20th century, no complete description of its theoretical behaviour appears to have been formalised.

Although banking system instability has been typically treated as a regulatory issue, with significant changes being made to the various frameworks over time as a result, the results shown here suggest that instability may be an intrinsic feature of many configurations ofthe system.

Although the simplified model discussed in this paper is not representative of current Basel systems, it is complex enough to offer some intuition about additional sources of instability in today's banking systems. However, without a detailed and precise description of day to day banking operations it is hard to be completely deterministic about the behaviour of any given instance of the system.

It is clear from public data for example, that the usual state of the system is more or less continuous expansion of the total quantity of bank deposits. This empirical result is in flat contradiction to the textbook prediction, which at best could be interpreted as supporting a periodic contraction and expansion due to loan repayment within known limits. However, since any loan default would trigger a loan and money supply contraction, and since loan defaults are also known to occur continuously, the more likely explanation for the empirically observed behaviour of the system is that it exists in a balance between mechanisms creating continuous deposit expansion and the monetary losses caused by continuous loan default.

The potential for monetary expansion purely due to race conditions in interbank lending is certainly one explanation for the system's expansionary tendencies, and one that would explain why this behaviour was also observed during the gold standard period. But there are almost certainly other mechanisms that can trigger inadvertant deposit expansion by the system. Basel regulated systems for example, allow limited forms of debt to be used in the equity capital holdings which are used to regulate the quantity of debt issued by the system[4]. The establishment of new banks would also be a source of deposit expansion, and banking systems vary quite widely in both the number of banks, and the ease with which new ones can be created.

It should not be overlooked that in a system that is this sensitive to what appears to be a variety of conditions, software problems may be a confounding issue. Ideally, construction of independent models of the system would allow direct comparisons of results to be made, which would assist in determining the precise causes of different behaviours.

As a critical component of money and credit regulation then, the mechanical behaviour of the banking system clearly warrents further study. Of particular interest are differences between regulatory frameworks, and aspects of the network elements of debt flows that can affect its behaviour, such as loan flows between banks, the interbank lending topology, and the number of banks in the banking system. One problem with the model presented here is that it does not attempt to reproduce the variability of loans, loan periods and interest rates in the general economy. Whether the superposition of heterogeneous loans would increase or decrease the instabilities described here is an open question. Berardi[2] has done some initial work on simulating systems with more varied components, but there is clearly much work still to be done.

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