The Nature of Thermodynamic Entropy

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Thermodynamic entropy is a pathway for transitioning from the mechanical world of fundamental physics theory to the world of probabilities, statistics, microstates, and information theory as applied to analyses of the universe. This paper proposes to explain the physical meaning of thermodynamic entropy, and, its partial connection to Boltzmann's entropy. The inclusion of Boltzmann's constant in his definition of entropy establishes the partial connection. The meaning of that connection between Clausius' entropy and Boltzmann's entropy is explained. These explanations first require new analyses of fundamental properties such as force and mass.

1 Introduction

The information contained here is original except for some basics of thermodynamics. The presentation has its theoretical basis originating in new theory incorporating fundamental unity from its start. This paper is not about presenting that theory. However, the premises are the same. The first premise is: There is a single cause for all effects. Theoretical multiple causes are reduced down to one. The second premise is: All properties inferred from empirical evidence must ultimately be expressible in the terms of that empirical evidence. The empirical evidence is always about patterns in changes of velocity. Historically, mass was arbitrarily chosen to be an indefinable property joining the naturally indefinable properties of space and time. This paper begins its analysis by reversing this practice. Mass is defined in terms of distance and duration only. The units of mass are formed from combinations of meters and seconds. Duration is cyclic activity used as a standard for mechanically measuring the passage of time.

Some introductory texts do not make clear that there is a difference between definable properties and indefinable properties. Yet, the problem is there for alert readers to spot. For example, distance and duration are defined in terms of natural

properties such as atomic frequency and the distance that light travels. However, the definition of the unit of measurement of mass is a human constructed object. Older texts used to make clear why the unit of mass is not definable in terms of a natural property. The reason is that mass is not a naturally indefinable property. An indefinable property is one that cannot be defined in terms of other pre-existing properties. Neither distance nor duration can be defined in terms of pre-existing properties. Yet they are the properties from which all empirical evidence is gathered. Therefore, the definitions of their units can be set as consistent measures of natural properties.

1.1 New Definitions for Mass and Force

The practice of distinguishing between definable and indefinable properties is here expanded upon. An example of the difference between definable and indefinable properties is: Mass is given units of kilograms. Kilograms are not definable in terms of meters and seconds. Therefore, mass was arbitrarily made into an indefinable property joining with the special status of distance and duration. After this arbitrary act is performed, force does become definable in terms of so-called pre-existing properties. Force is defined in terms of distance, duration and mass. Its units of newtons are defined in terms of meters, seconds and kilograms.

In addition to the naturally indefinable properties of distance and duration the patterns in empirical evidence make clear that there are at least two other properties, force and resistance to force. Those two properties are inferred from empirical evidence and must, therefore, be definable in terms of that empirical evidence. This means that both force and mass must ultimately be expressible in combinations of measures of distance and duration. The units of kilograms are unnecessary and detrimental to theory that makes use of the indefinable status assigned to mass. Fundamental unity is abandoned. When mass is defined in terms of distance and duration, force will also be definable in terms of distance and duration.

The equation $f/m=a$ shows that if force and mass are expressed in terms of distance and duration only, their units must reduce to those of acceleration. There are a few choices to try, however the one that works is for mass to have the units of inverse acceleration. The units of force would then be those of acceleration divided by those of acceleration. It would be a ratio of like properties. The empirical units, consisting of meters and seconds, cancel and leave force without units. Force will still have a magnitude and units of newtons can be retained for convenience of communication. This inverse acceleration definition for mass must refer to a real fundamental property. A premise of this paper is that there is only one physical origin for all

effects. The acceleration in the denominator of mass is taken to be representative of this single universal cause for all effects.

2 Electric Charge and Time

A premise presented in this paper is that the single original cause is represented by the acceleration that appears in the denominator of mass. Electric charge is a universal constant and is not representative of a property that undergoes acceleration. In the theory from which this work comes, each theoretical cause is representative of a particular aspect of the single original cause. There is also the premise that all properties inferred from empirical evidence must be expressible in terms of distance and duration. It must be determined how to represent electric charge in terms of distance and/or duration. Its units must be defined in terms of meters and/or seconds.

Electric charge is a very important fundamental constant and the speed of light is a very important fundamental property. The units for the speed of light are meters/sec. There is an anomalous relationship between electric charge and the speed of light:

$$eC = 4.8x10^{-11} \text{ ?}$$

This magnitude is the size of the radius of the hydrogen atom. Therefore, I take advantage of the anomaly and test new units for electric charge. I use the hydrogen atom to test the concept that electric charge should have empirical units of seconds. I will use Δx_c to represent the radius of the hydrogen atom:

$$\Delta x_c = eC = 4.8x10^{-11} meters$$

Making this kind of change to the definition of electric charge cannot be rigorously defended in this paper. The manner in which electromagnetic effects are accounted for is not explained here. The goal of this paper is to explain thermodynamic entropy. The results are considered to be convincing. I let electric charge represent a universally constant measure of duration. It becomes convenient to use the word 'time' to refer to duration. Therefore, instead of electron charge being represented as:

$$e = 1.602x10^{-19} \text{ coulombs}$$

It will be represented as a period of time by:

$$e = 1.602x10^{-19} \text{ seconds}$$

I use the value of electric charge from the *mks* system of units. I found that systems of units that define electric charge using Coulomb's equation setting the proportionality constant to unity give a corrupted value. In the *mks* system, electric charge is defined by referencing it to measures of force and distance, two true empirical properties. The proportionality constant must be determined by experiment.

The meaning of Coulomb's equation is revealed through that constant. That meaning is not included here.

3 Thermodynamic Entropy

If the premises set for this paper are accepted, at least out of interest, then a clear physical meaning can be offered for thermodynamic entropy. The explanation begins with Planck's constant.

3.1 Planck's Constant .

Planck's constant is the proportionality constant relating energy to frequency:

$$E = h\omega$$

Force times distance yields units of energy as meters. The units of frequency remain inverse seconds. Therefore, the units of Planck's constant are meters times seconds.

3.2 Boltzmann's Constant .

There is an analogous relationship between force and frequency. To show this I begin with:

$$h = \frac{E}{\omega}$$

For the hydrogen atom, the first energy level can be represented as an incremental value:

$$E_c = \Delta E$$

So, I write Planck's equation as:

$$h = \frac{\Delta E}{\omega}$$

Dividing both sides by the incremental Δx_c:

$$\frac{h}{\Delta x_c} = \frac{1}{\omega}\frac{\Delta E}{\Delta x_c}$$

Since:

$$f = \frac{\Delta E}{\Delta x_c}$$

Then:

$$\frac{h}{\Delta x_c} = \frac{f}{\omega}$$

Solving for force:

$$f = \frac{h}{\Delta x_c} \omega$$

To divide Planck's constant h by the radius of the hydrogen atom, I make use of the equality:

$$\Delta x_c = v_c \Delta t_c$$

Substituting:

$$f = \frac{h}{\Delta x_c} \omega = \frac{h}{v_c \Delta t_c} \omega$$

In the terms of current modern physics, this equation is analogous to:

$$f = \frac{h}{Ce} \omega$$

Where C is the speed of light and e is electric charge. Substituting the appropriate values:

$$f = \frac{h}{\Delta x_c} \omega = \frac{(6.625x10^{-34} \; meters \cdot sec)\omega}{\left(2.998x10^8 \; \frac{meters}{sec}\right)(1.602x10^{-19} \; sec)} = (1.38x10^{-23} sec)\omega$$

I used the empirical units of this paper. Even so, the value of the constant of proportionality is very recognizable. Its magnitude is the same as that of Boltzmann's constant. I offer the result that:

$$f = k\omega$$

Where k represents Boltzmann's constant. I did not include my usual subscripts because I want to show it in a form consistent with Planck's well-known energy as a function of frequency equation given before it.

3.3 Momentum and Frequency .

There can also be shown a relationship between photon momentum and frequency. I will solve for the proportionality constant of this relationship. Force is defined as:

$$f = \frac{\Delta P}{\Delta t_c}$$

Then, using the equation derived two steps above, I write:

$$k = \frac{f}{\omega} = \frac{1}{\omega}\frac{\Delta P}{\Delta t_c}$$

Where k is Boltzmann's constant. Solving for momentum:

$$\Delta P = k\Delta t_c \omega$$

The proportionality constant is:

$$k\Delta t_c = (1.38x10^{-23}\,seconds)(1.602x10^{-19}\,seconds) = 2.21x10^{-42}\,seconds^2$$

Introducing a symbol b for this constant:

$$b = k\Delta t_c = \frac{h}{\Delta x_c}\Delta t_c = \frac{h}{v_c} = 2.21x10^{-42}\,seconds^2$$

The relationship can then be generalized as:

$$P = b\omega$$

3.4 Temperature and Frequency .

It is of theoretical importance that Boltzmann's constant appears to be a part of the frequency relationships discussed above. This occurs because the preceding sections demonstrated that:

$$h = k\Delta x_c$$

This relationship allows me to mix formulas that contain either of these constants. For example, I can investigate the possible theoretical meaning of equating:

$$E = \frac{3}{2}kT$$

With:

$$E = h\omega$$

Combining these equations yields:

$$\frac{3}{2}kT = h\omega$$

Rearranging terms:

$$\frac{T}{\omega} = \frac{2}{3}\frac{h}{k}$$

Substituting:

$$\frac{T}{\omega} = \frac{2}{3}\Delta x_c$$

Solving for T and substituting for ω:

$$T = \frac{2}{3}\Delta x_c\,\omega = \frac{2}{3}\Delta x_c\frac{E}{h} = \frac{2}{3}\frac{E}{h/_{\Delta x_c}} = \frac{2}{3}\frac{E}{k}$$

Energy is given empirical units of meters. Planck's constant then has units of meters times seconds. The units of temperature must be those of meters divided by seconds. The use of meters in temperature represents the units of energy. Temperature is the rate of propagation of kinetic energy between gas atoms. Substituting the magnitude of the radius of the hydrogen atom:

$$T = \frac{2}{3}(4.8x10^{-11}meters)\frac{E}{h} = \frac{2}{3}(4.8x10^{-11}meters)\omega$$

Yielding the relationship:

$$T = (3.2x10^{-11}meters)\omega$$

3.5 The Physical Meaning of Thermodynamic Entropy

Temperature is an integral part of the definition of thermodynamic entropy and must be clearly, correctly, and fully defined. The typical explanation offered that temperature is a measure of average molecular kinetic energy does not fulfill these requirements. A new interpretation was presented above. Temperature is a measure of the rate of exchange of kinetic energy per molecule. Using this interpretation, the following is the explanation of thermodynamic entropy. Thermodynamic entropy was discovered long before microstates were discovered. Boltzmann's later definition of entropy, using 'phase space cells' and then microstates, included Boltzmann's constant from ideal gas theory, a classically derived constant with units of joules/°Kelvin. These units become, in this paper, the empirical units of meters divided by meters per second, reducing down to seconds.

Clausius' definition of thermodynamic entropy is precise. It is defined under ideal conditions. It is not a process of molecules rearranging themselves at non-equilibrium rates. It does not include any system for which any part can vary from its average temperature. It does not refer to a general process of achieving thermal equilibrium. There are literally no conditions of disequilibrium. We very closely approximate it by restricting the analysis to infinitesimal changes. Entropy is defined as a mathematical function representing an ideal relationship of the transfer of energy into or out of a system at constant temperature. Energy in transit is called heat Q. Thermodynamic entropy's definition is:

$$\Delta S = \frac{\Delta Q}{T}$$

Where ΔS is a change in entropy, ΔQ is the transfer of an increment of energy either into or out of a system, and T is the temperature of the system in degrees Kelvin. This equation is based upon a Carnot engine. In the differential form, the equations for a series of Carnot engines accurately represent a continuous path on a generalized work diagram so long as the series of engines represented are quasi-static, i.e. no dissipative effects, and reversible, i.e. returns to initial conditions at the end of each cycle. The differential forms of these equations may be solved by means of calculus for the changes in entropy.

The classical definition of entropy, expressed in terms of macroscopic properties, shows how entropy is calculated, but does not make clear what entropy is. It is a mathematical expression and not an explained physics property. It is temperature that masks the identity of entropy. Temperature is an indefinable property in theoretical physics. It is a fundamentally unique indefinable property along with distance, time, mass, and electric charge. It is assigned indefinable units of measurement called 'degrees'. If the physical empirical action that is temperature was identified then entropy would be explained. The new definition given above for temperature will now be used to explain the physical meaning of thermodynamic entropy.

What is thermodynamic entropy? It is something whose nature should be easily established, because, its derivation is part of the operation of the simple Carnot engine. In fact the answer can be found in the operation of the Carnot engine. The Carnot engine is the most efficient engine, theoretically speaking. Its efficiency is independent of the nature of the working medium, in this case a gas. The efficiency depends only upon the values of the high and low temperatures in degrees Kelvin. Degrees Kelvin must be used because the Kelvin temperature scale is derived based upon the Carnot cycle. The engine's equation of efficiency and the definition of the Kelvin temperature scale are the basis for the derivation of the equation:

$$\frac{Q_{high}}{T_{high}} = \frac{Q_{low}}{T_{low}}$$

Something very important happens during this derivation that establishes a definite rate of operation of the Carnot cycle. The engine is defined as operating quasi-statically. The general requirement for this to be true is that the engine should operate so slowly that the temperature of the working medium should always measure the same at any point within the medium. This is a condition that must be met for a system to be described as operating infinitesimally close to equilibrium.

There are a number of rates of operation that will satisfy this condition; however, there is one specific rate, above which, the equilibrium will be lost. Any slower rate will work fine. The question is: What is this rate of operation that separates equilibrium from disequilibrium? It is important to know this because it is the rate that becomes fixed into the derivation of the Carnot engine. This occurs because the engine is defined such that the ratio of its heat absorbed to its heat rejected equals the ratio of the temperatures of the high and low heat sources:

$$\frac{Q_{high}}{Q_{low}} = \frac{T_{high}}{T_{low}}$$

Temperature is proportional to the rate of exchange of energy between molecules. It is not quantitatively the same as the rate, because, temperature is assigned arbitrary units of measurement that are not time, distance, or a combination of these two. Temperature is assigned the units of degrees Kelvin and its scale is arbitrarily fitted to the freezing and boiling points of water. The temperature difference between these points on the Kelvin scale is set at 100 degrees. For this reason, the quantitative measurement of temperature is not the same as the quantitative measurement of exchange of energy between molecules. However, this discrepancy can be moderated with the introduction of a constant of proportionality k_T :

$$\frac{dQ}{dt} = k_T T$$

The ratio *dQ/dt* is the definition of the modified temperature. Multiplying by *dt*:

$$dQ = k_T dt \, T$$

Comparing this equation to the definition of thermodynamic entropy indicates that the differential of entropy appears in the above equation as:

$$dS = k_T dt$$

Both *dS* and *dt* are variables. It is necessary to determine a value for the constant k_T. This value may be contained in the ideal gas law:

$$E = n\frac{3}{2}kT$$

Where k is Boltzmann's constant. If I let $n=1$, then the equation gives the kinetic energy of a single molecule. In this case E becomes ΔE an incremental value of energy:

$$\Delta E = \frac{3}{2}kT$$

This suggests that for an ideal gas molecule:

$$\Delta S = \frac{\Delta E}{T} = \frac{3}{2}k$$

In other words, the thermodynamic entropy of a single ideal gas molecule is a constant. Substituting for Boltzmann's constant:

$$\Delta S = \frac{3}{2}\left(1.38x10^{-23} \ \frac{joules}{molecule \cdot °Kelvin}\right) = 2.07x10^{-23} \ \frac{joules}{molecule \cdot °Kelvin}$$

I have defined Entropy as:

$$\Delta S = k_T \Delta t$$

Therefore, I write:

$$k_T \Delta t = 2.07x10^{-23} \ \frac{joules}{molecule \cdot °Kelvin}$$

If I could establish a value for Δt, then I could calculate k_T. Since this calculation is assumed to apply to a single gas molecule and is a constant value, I assume that in this special case, Δt is a fundamental increment of time. There is a fundamental increment of time previously introduced in this paper. It is:

$$\Delta t_c = 1.602x10^{-19} \ seconds$$

Substituting this value and solving for k_T:

$$k_T = \frac{2.07x10^{-23} \ \frac{joules}{molecule \cdot °Kelvin}}{1.602x10^{-19} \ sec} = 1.292x10^{-4} \ \frac{joules}{molecule \cdot sec \cdot °Kelvin}$$

Substituting empirical units, previously presented, for each quantity and dropping the molecule indicator:

$$k_T = 1.292x10^{-4}$$

The value k_T is a unit free constant of proportionality. It follows that Boltzmann's constant is defined as:

$$k = \frac{2}{3} \Delta S$$

For the ideal gas equation, the thermodynamic entropy of each molecule is a constant:

$$\Delta S = k_T \, \Delta t_c$$

However, thermodynamic entropy is a property of an aggregate of molecules. As pressure increases, molecular interactions also increase. I have a value for the constant Δt_c but, the increment of time, as pressure increases, is not a constant. There are a great number of molecules involved and their interactions combine together. The increment of time becomes a variable. I expand the meaning of entropy into a more general form and substitute k_T into the general thermodynamic definition of entropy:

$$\Delta S = k_T \, \Delta t$$

The Δt in this equation is not the same as the Δt_c in the equation for a single molecule. In the macroscopic version, it is the time required for a quantity of energy, in the form of heat, to be transferred at the rate represented by the temperature in degrees Kelvin. Substituting this equation for entropy into the general energy equation:

$$\Delta E = \Delta S \, T = k_T \, \Delta t \, T$$

Recognizing that the increment of energy represents an increment of heat ΔQ entering or leaving the engine, and solving for ΔS:

$$\Delta S = \frac{\Delta E}{T} = \frac{\Delta Q}{T} = k_T \, \Delta t$$

Solving for Δt:

$$\Delta t = \frac{\Delta S}{k_T} = \frac{\Delta Q}{k_T T}$$

This function of Δt is what would have become defined as the function of entropy if temperature had been defined directly as the rate of transfer of energy between molecules. The arbitrary definition of temperature makes it necessary to include the proportionality constant k_T in a definition of modified thermodynamic entropy ΔS_m. Writing an equation to show this:

$$\Delta S_m = \frac{\Delta Q}{k_T T} = \frac{\Delta Q}{\Delta Q / \Delta t}$$

In particular, for an ideal gas receiving energy from a high temperature reservoir:

$$\Delta S_m = \frac{\Delta Q_{high}}{k_T T_{high}} = \frac{\Delta Q_{high}}{\frac{\Delta Q_{high}}{\Delta t}}$$

For a Carnot engine:

$$\Delta S_m = \frac{\Delta Q_{high}}{k_T T_{high}} = \frac{\Delta Q_{low}}{k_T T_{low}}$$

Therefore:

$$\Delta S_m = \frac{\Delta Q_{high}}{\frac{\Delta Q_{high}}{\Delta t}} = \frac{\Delta Q_{low}}{\frac{\Delta Q_{low}}{\Delta t}}$$

And the increments of time must be equivalent. This is why the increase in entropy is exactly the opposite of the decrease in entropy for the Carnot engine. The increment of heat entering the engine carries the positive sign, and the increment of energy leaving the engine carries the negative sign. Temperature is proportional to average kinetic energy because, it is proportional to the rate at which average kinetic energy is transferred between individual molecules. The numerator of modified temperature is average kinetic energy. Modified temperature establishes the point where equilibrium exists. Equilibrium exists when kinetic energy exchanges at the rate of modified temperature or lower.

Now, I consider an engine that operates infinitesimally close to equilibrium conditions, but has heat loss that does not result in work. The heat that is successfully converted into work can be represented by a series of Carnot engines. For the series of Carnot engines, the change in entropy per cycle is zero. The lost heat can be treated as if it just passes through the engines unnoticed. The series of engines is an unaffected pathway for the lost heat to travel from the high heat source to the low heat source. The lost heat becomes energy no longer available for producing work by the series of Carnot engines. The entropies that are affected are those of the high heat source and the low heat source. The entropies are measures of time required for the lost heat to be released by the high heat source and later absorbed by the low heat source. The net change in entropy is:

$$\Delta S = \frac{Q_{lost}}{T_{low}} - \frac{Q_{lost}}{T_{high}} = \Delta S_m k_T$$

The quantity of heat transferred is the same in both cases. The rates at which that heat will be transferred are different. The low temperature represents a slower rate of exchange of heat than for the high temperature. This means it takes longer for the low temperature source to absorb the quantity of lost heat than it does for the high

temperature source to emit the heat. This time difference is the change that occurs and it is proportional to the measure of change of entropy. The high heat source loses entropy because it requires extra time for the lost heat to leave the source. The low heat source gains entropy because it requires extra time to absorb the heat that is simply passing through the engine without being converted into work. This time difference is what is calculated as thermodynamic entropy. Thermodynamic entropy, referred to as an arrow of time, is an arrow of time.

4 Boltzmann's Entropy

The reason for defining thermodynamic entropy using an ideal gas is that as the pressure approaches zero, the exchanges of energy between molecules theoretically reduce down to single exchanges, one at a time without delay. That is the ideal representation for an ideal gas. Thermodynamic entropy applies to that pressure where the exchanges that occur can be ideally represented by each molecule taking its turn to pass on a measure of average kinetic energy equal to the magnitude of the quantity of average kinetic energy in the numerator of the gas' modified temperature. This process can be accurately modeled by considering all of the gas molecules to be lined up and the kinetic energy of one of them is transferred down the line from molecule to molecule until the energy has been transferred to the last molecule. The time required to complete this process is thermodynamic entropy.

Temperature is proportional to the rate transfer of average molecular kinetic energy of a gas. Thermodynamic entropy for an ideal gas, after the temperature is modified as set forward in this paper, is the time required for energy to be introduced into the gas, from external individual molecule or molecules to an individual gas molecule, at a rate set by the modified temperature of the gas. The modified temperature is the rate at which energy is transferred internally or locally between molecules. The time represented by the thermodynamic entropy of an ideal gas is the time required for each individual molecule to transfer its kinetic energy to another molecule, in turn, until all transfers are completed.

The universal gas constant R is defined as the pressure times volume of a mole of gas divided by the temperature of the triple point of water defined as 273.16 degrees Kelvin. It is usually written as:

$$pV = RT$$

Below, the pressure times volume, according to the kinetic theory of gases, is equal to the universal gas constant times temperature and to 2/3 the internal kinetic energy U of N molecules of gas:

$$pV = RT = NkT = N\frac{1}{2}m\overline{v^2} = \frac{2}{3}U$$

The average molecular kinetic energy divided by modified temperature is equal to the time period, as defined in this paper, required for the increment of energy to be transferred:

$$\frac{\frac{1}{2}m\overline{v^2}}{k_T T} = \Delta t_c$$

Boltzmann's constant is defined, in this paper, as the first equal term and by thermodynamics as the second equal term:

$$k = \frac{2}{3}k_T \Delta t_c = \frac{R}{N_0}$$

Where N_0 is Avagodro's number. Solving for R :

$$R = \left(\frac{2}{3}\right) N_0 \, k_T \Delta t_c$$

Substituting the appropriate values:

$$R = \frac{2}{3}\left(6.02x10^{23}\,\frac{molecule}{mole}\right)(1.29x10^{-4})(1.6x10^{-19}sec) = 8.31\frac{molecule \cdot sec}{mole}$$

For one mole of gas and dropping the molecule indicator:

$$R = \left(\frac{2}{3}\right)(6.02x10^{23})(1.602x10^{-19}seconds) = 8.31\;seconds$$

The universal gas R constant is the total time required for a mole of ideal gas to transfer average internal kinetic energy from single molecule to single molecule without delay between exchanges at the rate set by its modified temperature until Avogadro's number N_0 of molecules is reached. Boltzmann's constant is the time period represented by the universal gas constant reduced to single molecule status:

$$k = \frac{R}{N_0} = \frac{8.31\;seconds}{6.02x10^{23}} = 1.38x10^{-23}seconds$$

Boltzmann's constant is a measure of time that is proportional to the period of time necessary for a single exchange of average kinetic energy to occur between two molecules of an ideal gas:

$$k = \left(\frac{2}{3}\right)k_T\Delta t_c = (8.61x10^{-5})\Delta t_c$$

The actual time period is given by:

$$\Delta t_c = \left(\frac{3}{2}\right)\frac{k}{k_T} = (1.602x10^{-19}\,seconds)$$

The number of possible arrangements for a mole of ideal gas is infinite. Boltzmann's entropy requires there to be a limited number of possible arrangements. His entropy assumed the volume of the mole of gas could be divided into a limited number of cells available to be occupied. In quantum theory, there are a natural limited number of available arrangements. Instead of arbitrary cells, there are microstates which particles might occupy. If the concept of microstates is idealized so that all microstates are equally likely to be occupied, then, I can write:

$$S = k\,\Omega$$

This is not the definition of Boltzmann's entropy even though Ω is the number of microstates. The inclusion of Boltzmann's constant causes this calculation to be analogous to that of thermodynamic entropy. The number of microstates simulates a number of ideal gas molecules. The entropy calculation simulates the calculation of internal entropy for a mole of ideal gas. The solution is proportional to the time period required for the simulated ideal gas molecules to transfer their individual average kinetic energies from one molecule to the next, without delay, until the number of simulated molecules equals Ω. The calculation of internal thermodynamic entropy for equivalent numbers of ideal gas molecules is the same for all temperatures. The calculation of the entropy represented by the above equation for a number of microstates equal to an equivalent number of ideal gas molecules will yield a solution identical to an analogous calculation for the equal number of ideal gas molecules. Boltzmann's entropy is:

$$S = k\,\log\Omega$$

Therefore, Boltzmann's entropy is proportional to the time period of a single transfer of ideal gas molecule energy times the logarithm of the number of microstates. The units of seconds carried along by Boltzmann's constant have become irrelevant. Boltzmann's constant can be set to unity without units. Its connection to thermodynamic entropy is already lost. The loss of this connection allows for entropy to represent whatever the theorist deems useful such as statistical entropy and information entropy.

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