Emergent Patterns in Dance Improvisation and Choreography

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In a traditional choreography a choreographer determines the motions of a dancer or a group of dancers. Information theory shows that there is a limit to the complexity that can be created in any given amount of time. This is true even when building on previous work, since movements and their interactions have to be communicated to the dancers. When creating a group work, choreographers circumvent this problem by focusing either on the movements of individual dancers (giving rise to intricate movements but within a simple spatiotemporal organization) or on the overall structure (intricate patterns but simple movements) or by creating room for the dancers to fill in part of the movements. Complexity theory offers a different paradigm towards the generation of enticing patterns. Flocks of birds or schools of fish for instance are considered 'beautiful' but lack a central governing agent. Computer simulations show that a few simple rules can give rise to the emergence of the kind of patterns seen in flocks or swarms. In these models individual agents are represented by dots or equivalent shapes. To be of use to choreography and to be implemented on or rather with dancers, some additional rules will therefore have to be introduced. A number of possible rules are presented, which were extracted from 'real life' experiments with dancers. The current framework for modeling flocking behavior, based on local interactions between single agents, will be extended to include more general forms of interaction. Dancers may for instance perceive the global structure they form, e.g. a line or a cluster, and then put that knowledge to creative use according to some pre-established rules, e.g. if there is a line, form a circle or if there is a cluster spread out in all directions. Some of these rules may be applied back to other complex systems. The present paper is also an invitation to complexity theorists working in different fields to contribute additional rules and ideas.

1 Introduction

A choreography is a set of instructions for the organization and reconfiguration of one or several bodies in space and time. In practice there is always a 'residual term' ε, in the sense of performance = choreography + ε, which is not covered by the explicit instructions and which is left to the dancer(s) to fill in. It follows that the more complex a choreography, the longer the set of instructions. This means that there is a limit to the degree of complexity that can be created by a choreographer in any given amount of time. In dance there are various ways of dealing with increasing complexity. A choreographer can concentrate on each dancer's individual movements and organize the dancers in simple patterns, such as lines, circles and clusters, s/he can create intricate global patterns while keeping the individual movements simple or increase what I called the residual term, leaving more of the individual movements to the dancers to fill in.

Complexity theory offers a different paradigm towards the generation of complex structures. Flocks of birds and schools of fish for instance, exhibit intricate patterns that emerge from the interaction of individual agents, in the absence of a 'master mind' or central governing agent. Computer simulations have shown that a few simple rules give rise to the kind of patterns seen in swarms [e.g. Reynolds 1987, Tone & Tu 1998, Csahók & Vicsek 1995, Czirok et al 1997], pedestrian [Helbing & Molnár 1995] and traffic flow [Helbing 2001] and large crowds of people [Helbing et al. 2000]. Another intriguing finding is that large groups of interacting agents or particles not only exhibit aggregation and swarming but also spontaneous synchronization [e.g. during the applause after a performance, Néda et al. 2000], an important aspect of the aesthetic appeal of choreographed dance. It therefore seems natural to try and translate these rules to dance.

A quick glance at the principles that have been incorporated in the models underlying these computer simulations shows that some can be made to depend on other, possibly hidden, variables. For instance, we could make distance to the nearest neighbor into a dependent variable, instead of a constant and invent our own criteria for relating inter-agent direction and velocity.

To make the transition from dots on a screen to multi-limbed dancers moving in three-dimensional space requires the design of additional rules and principles. After all we would like the dancers to do more than just walk around. In the present paper I will describe some of the rules I have designed to this end, for which I transformed a dance studio into a laboratory for studying complex systems.

2 What I Mean by Complexity

The present paper differs from other studies of complex systems in that I do not attempt to model an existing system, but rather try to *produce* a complex system. In doing so I loosely draw on the methodology sketched out by John Holland [Holland 1995 and 1998]. The work by Eric Bonabeau and Guy Theraulaz on ant behavior may also serve as an analogy. Having analyzed how ants collectively find the shortest path to a food source, they played around with the variables in their model and were thus able to derive a more optimal solution to a problem not yet discovered by nature [Bonabeau et al. 1999 and 2000]. This is what I do with models for flocking. The present work is therefore *inspired* by research into complex systems and its scientific claims are modest.

It will be clear that an *emergent choreography* as will be described here can be regarded as a complex system in the sense that it consists of multiple interacting components (dancers), the properties of which are not fully described by those of the individual components (dancers). Analyzing the isolated movements of an individual dancer will not bring us closer to an understanding of the choreography.

In information theory the complexity of an object is defined as the length of the shortest program, which generates the object. This alternative notion of complexity, originally proposed by the Russian mathematicians Ray Solomonoff and Andrei Kolmogorov and independently by the American mathematician Gregory Chaitin, is known as Kolmogorov complexity or algorithmic complexity. A problem with Kolmogorov complexity is that it cannot be computed, as it is based on considering all possible programs for generating the object. However, limiting the number of admissible programs to a certain class makes it possible to approximate an object's algorithmic complexity [Li & Vitanyi 1997].

In *The Quark and the Jaguar* Murray Gell-Mann gives a similar definition of complexity [Gell-Mann 1994]. Take the following three pictures [adapted from Gell-Mann 1994].

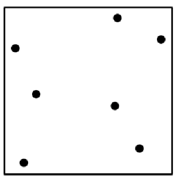

Figure 1a

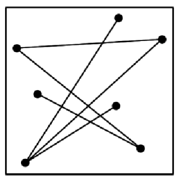

Figure 1b

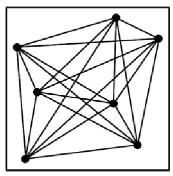

Figure 1c

Now which one is the most complex? None of the 7 dots in figure 1a are connected, so instructing someone over the phone to draw this image would require no more information than this, if we abstract from the position of the dots. In figure 1c all 7 dots are connected, and thus all we'd have to say would be 'draw 7 dots and connect all dots'. Now, if we wanted to transmit figure 1b we would have to specify exactly which dots are connected, and thus by the above definition, figure 1b is the most complex. This appears counterintuitive, because at first sight figure 1c looks the most complex. However, with 11 or 23 instead of 7 dots it would begin to look dense and from a distance there would seem to be little internal difference. Indeed it would matter little if there were 21 or 23 dots.

It's the same with movements of one or several bodies: constantly moving every conceivable part of the body may at first look complex, but is in fact quite simple: the task move every part of the body will produce a series of sequences that all look more or less the same.

The above example also illustrates the difficulty of quantifying the complexity of a choreography, whether a solo or a group work. We therefore have to rely on our senses and an estimate of what it would take to reproduce the ballet from a set of instructions. In principle one of the existing systems for dance notation, Labanotation or Benesh notation, could be used to this end. However, the fact that neither achieved popularity shows the difficulty of capturing dance in symbolic sequences. Perhaps the techniques that have been proposed to measure the complexity of RNA sequences may at some point be extended to dynamic sequences of body configurations [Adami & Cerf 2000].

3 Why Complexity Matters

The careful reader will have noticed that I use the term ‘complex’ not only as a way of characterizing a system, but also as a value or an objective, “complex=beautiful” or “complexity=great” The psychologist Mihalyi Cszikszentmihalyi has argued that the human brain actively searches for difference and more complex scenes or events, once it gets used to whatever it is currently doing or perceiving [Cszikszentmihalyi 1992]. If no such perceptual or cognitive challenges are at hand the brain will get bored. This may be why as they mature the work of many choreographers shows an increase in complexity, in a weak sense of algorithmic complexity.

In the 1930’s the mathematician George Birkhoff proposed a measure of beauty defined as M=O/C, whereby M stands for ‘aesthetic measure’ (or beauty), O for order and C for complexity [Birkhoff 1933, 1956]. In formulating this measure Birkhoff was inspired by the observation that beauty, whether in art or in nature, has something to do with order and complexity. Order shows in such compositional elements as repetition, contrast, symmetry, balance, synchronization etc. and is found in flowers, sculptures, figurative as well as abstract paintings, schools of fish, classical ballet and so on. Complexity Birkhoff defined in terms of the effort that goes into perceiving the object or event, an idea of renewed interest in the light of contemporary cognitive neuroscience. Birkhoff also referred to the definition of beauty by the 18th century Dutch philosopher Frans Hemsterhuis, “that which gives us the greatest number of ideas in the shortest space of time”. A perhaps unwanted consequence of his formula is that, as the degree of complexity goes to zero, and the degree of order is high, as in some abstract art, the measure of beauty goes to infinity. Its verbal formulation as ”the density of order relations in the aesthetic object” may therefore be a more appropriate description. In a recent paper Misha Koshelev and Vladik Kreinovich formalized Birkhoff’s idea by defining the complexity of an object in terms of its Kolmogorov complexity [Koshelev 1998, Kreinovich et al. 1998].

The various 20th century avant-gardes have shown the limitations of any approach to art that is purely based on an analysis of form. Recent progress in cognitive neuroscience shows that with respect to some art Birkhoff may have been on the right track [Zeki 1999, Ramachandran & Hirstein 1999] and it would be interesting to experimentally test his hypothesis. Not only scientific evidence supports Birkhoff’s claims. In a recent review of a mixed program by William Forsythe and the Frankfurt Ballet [1], a Dutch dance critic wrote that “Forsythe’s dance language is crystal clear, but of such a complex virtuosity [2] that the images and thoughts it evokes can hardly fall into place within the duration of a performance” [Rietstap 2002]. This almost reads as a reformulation of Frans Hemsterhuis’ and George Birkhoff’s definition of beauty. The critic concluded that “Forsythe’s dance language [is] too complex for [the] audience”, which would be explained by the gap between a choreographer with a career spanning over 20 years and an audience which sees a few dance performances a year.

4 Methods

In principle patterns also arise when particles or agents of whatever kind move randomly. Pattern perception is further enhanced by the tendency of the human brain to actively search for patterns and regularities [Ramachandran & Hirstein 1999]. Nonetheless in a purely random environment patterns are rare. One way of characterizing the present approach to dance and choreography is that it attempts to increase the likelihood of the emergence of patterns. We could, in mathematical terms, change the process from which the movements are drawn. This we could do by severely restraining the state space of possible movements. If all the dancers can do is move their arms up and down and walk back and forth, the dancers' movements and positions in space are more likely to momentarily synchronize.

Another possibility would be to correlate the dancers' movements, body configurations and positions in space. That is the approach described here. The idea is to describe the situations or stimuli a dancer can encounter and list the responses available to a dancer in a given situation [Holland 1995 and 1998]. As a meta-rule a dancer can either ignore a stimulus or use it as a cue for a motor response.

The present approach to dance and choreography relies on the dancers' ability to read each other's intentions. It also requires the dancers to understand the different rules and the patterns they give rise to. While being 'inside' the group they have to know how it will look from the outside. That is, the group has to develop a form of collective intelligence, with my own aesthetic preferences or more general principles of aesthetic experience, as a utility function that the group as a whole seeks to maximize [Wolpert & Tumer 1999]. This is why I work a lot with video feedback so the dancers can observe and learn from the local and global effects of their decisions. Technically speaking this is an example of reinforcement learning whereby my verdict and that of the dancers serve as reward signals.

The method I use is to extract rules from the observation of existing choreographed dance performances and other forms of human interaction and to re-apply those in a setting based on improvisation. I then observe the resulting behavior over a series of 'trials' to evaluate the emerging patterns and to determine where a conflict or a 'decision void' arises, that would require the introduction of additional rules. The work is therefore inherently inexact. Dancers may not stretch a rule to its limits, the way a computer model does, they may implicitly incorporate another rule and the number of trials may be too small to show all possible configurations. The present work may therefore serve as inspiration for more formal modeling approaches in collective robotics. It is also inevitable that my own preferences as to what I would like to see emerge on stage make themselves felt in the choice of rules. This is not per se a compromise as long as it is acknowledged.

5 A Selection of Rules

5.1 Spatial organization

Dancers need a motivation to go from A to B and to do one thing rather another. One of the simplest rules for moving through space is a random walk. This translates into the following instructions:

1. just walk around, at any moment you can turn into another direction
2. if you bump into a wall, another person or leave the stage, turn and continue in any direction
3. at any moment you can stop and stand still

From a distance and with a little imagination applying these rules gives the impression of a room filled with randomly moving particles. However, these rules are too general. Should a dancer only move forwards or is she allowed to also move backwards and sideways? This too has to be made explicit.

4. you can move forward, backward or sideways.

The above is just a basic setting to build on. The following rules add some interaction.

5. you can decide to trail another dancer (by walking behind or next to the other person)
6. if somebody is trailing you, you can try to escape by changing direction or speed
7. if you are trailing another person you can overtake that person
8. you can block another person by standing in front of that person

While these simple rules give rise to interesting patterns in terms of spatial organization they turn out to be insufficient for two reasons. On the one hand dancers tend to find them too unspecific, they don't say anything about the use of the arms or ways of walking, on the other hand it may take too long before certain desired patterns emerge. I therefore decided to introduce some specific rules for spatial organization, making use of the fact that human beings are able to look at a situation from different degrees of locality (from nearest neighbor to the entire field of view) and act accordingly.

It should be noted that most individual based models are based on local interactions: each agent only responds to the agents "within some fixed, finite distance which is assumed to be much less than the size of the 'flock'" [Toner and Tu 1998, p. 4828]. This rule is self-reinforcing: within a flock only the nearest neighbors are visible. If the distance between agents increases, other agents become visible, thus enabling long ranged interaction. As more agents become visible the potential for interaction in terms of the number of candidates also grows, but the likelihood of communication or mutual interaction, whereby dancer A interacts with dancer B and vice versa, decreases.

5.2 Alignment

Various species of caterpillars form queues when traversing an open space. Human beings too self organize into queues in front of check outs, box offices etc. While this is a cultural phenomenon, the phenomenon itself is no less real. One could think of various structures into which a group of people can self-organize, the simplest being alignment: standing side by side in a row, face to back in a queue, a circle, a square etc. (see figure 2).

5.3 Clustering

In computer simulations of complex systems (schools, herds, swarms etc.) the rules are chosen so that they give rise to spontaneous clustering (e.g. separation, alignment and cohesion, Reynolds 1987). Effectively when choreographing a group or 'block' of dancers, the dancers implicitly apply similar rules, because a choreographer will only indicate how and where s/he wants the *group* to move, and not each individual dancer. In the present context I have adopted the now classic rules for group formation to control *the movement of* a cluster. A problem from the point of view of dance improvisation is that with these rules and in the absence of external perturbations, dancers would tend to remain in a cluster. I therefore introduced specific rules to have a cluster emerge and dissolve. The first rule tells the dancers to cluster together if they see the core of a cluster forming (see figure 3a). The rule 'expand' tells the dancers to expand in all or one particular direction if a cluster has formed (see figure 3b). A cluster can also dissolve if dancers stop adhering to the group cohesion rule.

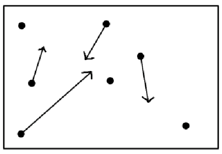

Figure 2

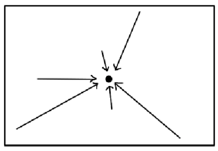

Figure 3a.

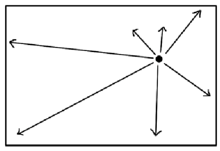

Figure 3b.

5.4 Levels in space

The above rules concern the layout or the map in space. To also control the organization of levels in space I designed the following rule. For simplicity I will assume there are three basic positions standing, sitting and lying on the floor. With three dancers, and not distinguishing between individual dancers, we then have 10 possible configurations (all three dancers standing, two sitting, one lying on the floor, one sitting, two standing etc). The idea is that each dancer knows all three positions and can adjust his position according to the position of the other dancers. Thus if all three dancers are standing up one dancer can decide to either sit or lie down on the floor. If two or three dancers have the same idea at the same moment in time we have spontaneous synchronization. If one dancer is lying on the floor and two others are standing, one can decide to also lie down on the floor whereupon the dancer already lying on the floor can decide to sit up, get up or keep lying on the floor.

5.5 Divisions of space

The above principle of adjusting the spatial configuration in relation to what the other dancers are doing is a simple and efficient means of creating spatial dynamics. It can be extended to the more general use of levels and regions (left, right, front, back) in space. With an understanding of spatial organization dancers can self-organize on the far left of the stage so as not to be constantly all over the stage. When observing a cluster a dancer can also create a contrast between group and individual by suddenly moving away. In general the space can be divided into various sections and according to different principles (a checker board, a soccer field etc.). It is then possible to devise all kinds of rules or 'games' for going from one section to another, all in relation to what another dancer is doing.

Example: Imagine You Are in a Labyrinth

For one particular piece I asked the dancers to imagine they are in a labyrinth. The dancers can only move back and forth and have to make rectangular turns. Because the corridors are narrow, to move the arms sideways they have to make a 90º turn. If two dancers meet they have to pass while adhering to the constraints imposed by the narrow corridors and the opportunities offered by corners [3].

5.6 Dynamics

The emphasis in the rules introduced so far is on spatial organization. The rules that govern the behavior of swarms also include velocity alignment. This we can translate to dance, but to make things a little more interesting and to avoid everybody dancing at the same speed, instead of adopting the same velocity we can introduce a range of options. If dancer A moves with a certain velocity, dancer B can decide to move slower, faster or with the same velocity or she can decide to freeze into a pose. Again the idea is that once the range of options available to a generic dancer have been established, they can be made interdependent.

5.7 Copy

It is also possible to directly link the movements of two or more dancers. If dancer A sees dancer B perform a movement (x) she can instantly copy the movement. Thus if dancer B raises her left arm above her head, so does dancer A, whatever the position she is in. She may be kneeling, sitting or squatting on the floor whereas dancer B is standing up. It is interesting to observe that this rule works best in non-neutral positions because it is easier to infer what the movement is going to be. If dancer A and B face each other, copying can have two meanings: if dancer B raises her left arm, dancer A either raises her left arm as well, putting herself in the shoes of dancer B, or she raises her right arm, to form a 'mirror image' of dancer B's movement. We could thus distinguish between 'copying' and 'mirroring'.

Corollary 1

If dancer A sees dancer B perform movement x_1 of a previously rehearsed movement sequence (x_1,..., x_n) she will know how the movement sequence will unfold and thus be able to better synchronize her movements. If she sees dancer B perform movement x_{n-m} she can perform the series (x_{n-m+1},..., x_n).

Corollary 2

Obviously movements can only be copied if they are visible. If dancer A happens to be standing behind dancer B, dancer B won't be able to copy dancer A's movements. Unless that is, we introduce a third dancer C, who is able to see and copy dancer A's movements and is visible to dancer B (see figure 4).

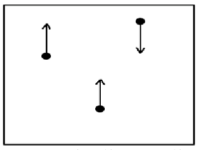

Figure 4. Arrows indicate direction of gaze

Corollary 3

If dancer A sees dancer B perform movement x_i of a previously rehearsed sequence (x_1,..., x_n) she can deliberately perform the movement sequence with a delay to create a 'ripple' effect, which becomes even stronger if dancer C copies dancer A with a delay. Dancer C may already have observed 'dancer A copying dancer B', making it easier for dancer C to copy the movement sequence.

Corollary 4

Instead of copying the movement of dancer B, dancer A can also use it as a cue for another movement, which can be either a previously rehearsed movement, a transformation of the original movement or in principle, 'any movement whatsoever', in which case the visible relationship with the original is lost. The difference between 'copying' and 'mirroring' when two dancers are facing each other can be regarded as a transformation of the original movement: left becomes right and vice versa. Such a left/right reversal can also be applied deliberately e.g. if dancer A is standing behind dancer B.

We thus have the following three possibilities:

1. IF x THEN x *(same movement)*
2. IF x THEN $f(x)$ *(different movement which relates to original movement)*
3. IF x THEN y *(different movement)*

The effect of this principle is the spontaneous synchronization of 2 or more dancers across space. It looks 'choreographed' but it isn't. The synchronization emerges from the interaction of the dancers.

5.8 Internal differentiation

One of the great discoveries in the modeling of crowds is that in large enough groups, individuals can be treated as identical agents. In other words flocks, herds and crowds form a homogeneous mass. A *small* group of agents on the other hand, can be differentiated into a heterogeneous group, for instance by having the agents wear different colored outfits. Division of a group into two or more subgroups introduces novel opportunities for interaction. The fact that the 22 players in soccer are divided into two teams of 11 players each, in combination with the spatial layout, determines the behavior of the agents and the patterns that emerge. An interesting phenomenon in soccer is that children all around the world always flock around the ball. In professional soccer field players are differentiated into (left/right) defenders, strikers and midfielders. Each player has a designated area and role within the field and the team's strategy. In choreography we could again invent our own rules.

5.9 Equivalence relationships

An equivalence class is a set of elements defined by an equivalence relation. Examples of equivalence relations are "is parallel to", "has the same color", and perhaps a bit more daring, "has the same meaning" and "has the same perceptual effect". In my work I am trying to find different equivalence relations and equivalence classes as they apply to dance. For instance it may matter little if a dancer turns left or right or if a position is performed in a left or a right orientation. The question the dancers and I then try to address is when it *does* matter. Similarly in a duet it may matter little which person performs which role, although again the question is when it does matter.

The more interesting question is what dancer A has to do to maintain an equivalence relation relative to another movement taken as the original movement, if dancer B performs a movement in a different orientation. To give a more concrete example, the combination of dancer A standing up and dancer B lying on the floor can be equivalent to dancer A lying on the floor and dancer B standing up. So, if two dancers have rehearsed a duet, then knowing various equivalence relations pertaining to that duet, they can recognize a situation and then perform an equivalent duet based on that situation. In the previous example dancer B may notice that dancer A is lying on the floor and then enter into a duet which had been rehearsed with dancer B lying on the floor and dancer A standing up.

Now the following two constellations may constitute a perceptual equivalence class in the sense that they have a similar perceptual effect on an observer:

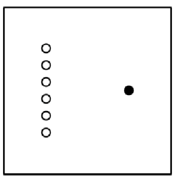

Figure 5a

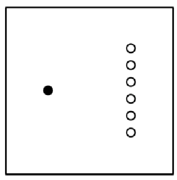

Figure 5b

In practice this means that if the dancer represented by the black dot is on the right side, all other dancers should go to the left to constitute constellation 5a, if however the dancer in black is on the left side of stage, the other dancers should align on the right side. Now, the next step is that any dancer can be the black dot. The dancers may notice that they can self-organize into either one constellation or a dancer may isolate him or herself and try to get the other dancers to align into figure 5b.

By defining a number of constellations in combination with a set of equivalence relations it is possible to direct the patterns that emerge on stage, without setting the piece as a whole. This approach has the additional benefit of creating a form of consistency across different performances while simultaneously allowing for novelty. One has, in other words, structure with variations, one of the characteristics of a complex system.

This form of interaction whereby the patterns stay the same, but the agents take turns, one finds not only in team sports such as soccer or basketball, but also in social animals that hunt as groups such as hyenas and wolves. Both species are known to chase their prey in a relay hunt and to drive an animal into an ambush formed by a (sub) group. This requires the predators to interact in response to what the others are doing. The chase itself is of course also a form of interaction, with both predator and prey performing their genetically determined escape and pursuit strategies (e.g. the zig zag of hare), which can also be implemented in dance.

6 Conclusion and Directions for Future Research

The present article has described some of the rules I have designed to govern the interaction of a group of dancers. The primary goal of my research is artistic: to have fascinating patterns emerge within a group of dancers. Although I myself have the dancers improvise on stage as well, the above rules can also be used as a choreographic rapid prototyping engine. When recorded on video, desired patterns can be singled out and fixed to be included in a 'standard' choreography. The equivalence classes described above are an example of this approach. Some of the rules proposed here may already have been discovered by nature. For instance prairie dogs feed in groups and always have at least one animal standing on guard on its hind legs. It appears that the moment this animal joins the group another animal takes over its role as guard. This is equivalent to the rule for levels in space introduced above (5.4). In addition to its artistic merits the present approach to dance improvisation and choreography may therefore serve as an experimental setting for investigating forms of group interaction, such as the behavior of social animals, that at present are hard to implement in a computer simulation.

Acknowledgements: I would like to thank the dancers I have worked with in exploring and bringing to life the ideas presented here.

Notes

[1] The program included: *7 To 10 Passages* (2000), *The room as it was* (2002), *Double/Single* (2002) and *One flat thing, reproduced* (2000).

[2] The critic may have meant virtuoso complexity.

[3] The "labyrinth" section in Communications from the Lab (2004), see http://www.ivarhagendoorn.com for a video excerpt.

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